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Iterative solvers for image denoising with diffusion models: A comparative study

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ABSTRACT

In this paper we propose and compare the use of two iterative solvers using the Crank–Nicolson finite difference method, to address the task of image denoising via partial differential equations (PDEs) models such as Regularized Perona–Malik equation or C-model and Bazan model (Bilateral-filter-based model). The solvers which are considered in this paper are the Successive-over-Relaxation (SOR) and an advanced solver known as Hybrid Bi-Conjugate Gradient Stabilized (Hybrid BiCGStab) method. From numerical experiments, it is found that the Crank–Nicolson method with hybrid BiCGStab iterative solver produces better results and is more efficient than SOR and already existing, in terms of MSSIM and PSNR.

1. Introduction

Image processing plays an important role in medical imaging, remote sensing, military, computer graphics, molecular imaging, surveillance video, auto-piloting, etc. During the past two decades, image processing has attracted the attention of many mathematicians. One of the most fundamental tasks in image processing applications is that of image de-noising, which is a significant preprocessing step. Due to inherent limitations of acquisition devices or the existence of random interruptions in the medium, most images are naturally dissipated by noise. Typically, it is assumed that the degradation process for image denoising can be expressed using a mathematical model [1].

For a long time, partial differential equations (PDEs) are a very effective tool to obtain a denoised image from noisy image [2–4]. The linear diffusion process is the simplest and well considered PDE-based method [5]. But it fails to keep edges or textures in the denoised image as a result of the low-pass convolution [6]. In recent years, the applications of nonlinear PDEs are popular in the field of image restoration to address the image de-noising problem. Numerous nonlinear PDE models are proposed for this purpose [7,8]. In [6], Perona and Malik proposed a nonlinear PDE which consists of an inhomogeneous diffusion coefficient. This model is useful for smoothing, de-noising and detection of edges in (digital) image. However, Perona–Malik (P–M) model is an ill-posed model [9]. Hence to attain a well-posed model Catté et al. [10] proposed a regularization of the P–M model, known as regularized Perona–Malik model, in which they modified this model by convolving image with Gaussian kernel for the diffusion coefficient.

Recently, Tomasi and Manduchi proposed a bilateral filter for removing noise from images, which combines spatial and range filtering [11]. Bilateral filter depends on spatial difference and intensity difference. In bilateral filter, a pixel

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Fig. 1. Boat noise free image (Top Left), noisy image with $\sigma = 20$ (Top Right), restored image using Existing scheme [10] (Down left), SOR solver based proposed scheme (Down middle) and Hybrid BiCGStab solver based proposed scheme (Down Right).

is simply replaced by weighted mean of its neighbors having similar gray level values and weighting the distance to the reference pixel [12]. It has been adopted for several applications like medical imaging, image restoration, etc. [13,14]. This is a non-iterative technique, but only if size of spatial box is large enough and this cause for over smoothing. Hence to do denoising it should be able to make balance between edge preservation and spatial window size. It is known that anisotropic diffusion plays important role for image denoising. Therefore the combination of anisotropic diffusion and bilateral filter has attracted many researchers in this decade, to improve the quality of denoised or filtered image while preserving the edge information [15–17,13,14,18]. In this work we have adopted the Bazan model.

The above discussed theory related to PDE models is continuous in nature. However, in practical problems, the image is discrete and represented by a collection of pixels on a fixed equidistant grid. Moreover, in most PDE models, analytical solutions are not possible. Thus, the diffusion filter models must be discretized by proper numerical schemes to compute the solutions. The development of selected numerical technique for the PDE models is a vital factor of PDE-based approaches [19]. One popular numerical scheme, in this respect is the Crank–Nicolson scheme [20], which has some advantages over traditional schemes, as discussed in Section 1.1.

In addition to choosing a proper numerical scheme, the iterative approaches, which are used to solve the system of algebraic equations generating from the numerical discretization, are also important considering factors such as faster convergence and better denoising accuracy [21]. A useful solver in this respect which has not yet been explored for image denoising is the Hybrid Bi-Conjugate Gradient Stabilized solver (BiCGStab(2)) [22].

In this paper, our main intention is to study the usefulness of advance iterative solvers (e.g. Hybrid BiCGStab method) for the popular Crank–Nicolson (C–N) Numerical Scheme, for image de-noising, when the model equations are non-linear and discretized using implicit numerical scheme.

1.1. Related work

Till date only few finite difference schemes are available to solve these PDE models. Usually the explicit numerical schemes have been used to discretize the corresponding nonlinear diffusion models [23]. Because the explicit scheme has the limitations of instability and several iterations [23,19,24], an important alternative is to use the implicit numerical scheme e.g. Crank–Nicolson (C–N) or higher order accurate numerical methods, to solve these models to get better denoising accuracy [25,26]. The use of C–N method has advantages such as unconditional stability, second order accuracy in time and space, and its mathematical/computational competency to find the optimal time step size [20].

It is quite cumbersome to solve such systems using direct methods, because of the requirement of memory space and the huge CPU times. Hence, to achieve better accuracy and speed up the calculation, iterative solvers have been used. Again, the accuracy of a numerical method depends on the right choice of iterative solver, which is ultimately going to solve the algebraic system of equations originating from the finite difference discretization. The problem, however, is that it requires a good knowledge and proficiency to select the proper iterative solvers [21,27,28]. The Krylov subspace methods like