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A note on GARCH model identification

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Abstract

Financial returns are often modeled as autoregressive time series with innovations having conditional heteroscedastic variances, especially with GARCH processes. The conditional distribution in GARCH models is assumed to follow a parametric distribution. Typically, this error distribution is selected without justification. In this paper, we have applied the results of Thavaneswaran and Ghahramani [A. Thavaneswaran, M. Ghahramani, Applications of combining estimating functions, in: Proceedings of the International Sri Lankan Conference: Visions of Futuristic Methodologies, University of Peradeniya and Royal Melbourne Institute of Technology (RMIT), 2004, pp. 515–532] on identification of GARCH models to a number of financial data sets.

Keywords: GARCH; Model identification; Least absolute deviation; Least squares; Estimating functions

1. Introduction

The ARIMA time series model suggested by Box and Jenkins [1], has enjoyed fruitful applications in forecasting social, economic, engineering, foreign exchange, and stock problems. This model assumes that the future values of a time series have a clear and definite functional relationship with current, past values and white noise.

Recently there has been growing interest in using nonlinear time series models in finance and economics (see Granger [2] and Thavaneswaran et al. [3]). Many financial series, such as returns on stocks and foreign exchange rates, exhibit leptokurtosis and time-varying volatility. These two features have been the subject of extensive studies ever since Nicholls and Quinn [4], Engle [5], and Engle and Gonzalez-Rivera [6] reported them. Random coefficient autoregressive (RCA) models, (Nicholls and Quinn [4]), the autoregressive conditional heteroscedastic (ARCH) model, (Engle [5], Engle and Gonzalez-Rivera [6]) and its generalization, the GARCH model, (Bollerslev [7]) provide a convenient framework to study time-varying volatility in financial markets. Financial time series models for intra-day trading are typical example of random coefficient GARCH models.

In practice, a common assumption in applying GARCH models to financial data is that the return series is conditionally normally distributed. We shall refer to this as the normal GARCH model. It is well known that the normal GARCH model is part of the volatility clustering patterns typically exhibited in financial and economic time

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series. However, the kurtosis implied by the normal GARCH model tends to be far less than the sample kurtosis observed for most financial return series. For example, Bollerslev [7] finds evidence of conditional leptokurtosis in monthly S&P 500 Composite Index returns and advocates the use of the t-distribution. Thus, the nonnormal GARCH model is more appropriate with the large leptokurtosis typically observed in asset returns.

In this paper, we study model identification problems for GARCH models using a combination theorem given in Thavaneswaran and Thompson [8]. Using the combination theorem, the correlation between least squares (LS) estimating functions and least absolute deviation (LAD) estimation functions is obtained and it turns out to be the asymptotic correlation between the corresponding estimators. We end with a Conclusions section. For more details on applications of combining estimating functions, see Thavaneswaran and Ghahramani [9]. In the next section, we give a brief description of GARCH models.

1.1. GARCH models

Consider the general class of GARCH(p, q) models for the time series y_t where

$$y_t = \sqrt{h_t} Z_t, \tag{1.1}$$

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}, \tag{1.2}$$

where Z_t is a sequence of independent, identically distributed random variables with zero mean, unit variance. Let $u_t = y_t^2 - h_t$ be the martingale difference and let σ_u^2 be the variance of u_t. Then (1.1) and (1.2) could be written as:

$$y_t^2 - u_t = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j},$$

$$\phi(B) y_t^2 = \omega + \beta(B) u_t, \tag{1.3}$$

where, $\phi(B) = 1 - \sum_{i=1}^{r} \phi_i B^i$, $\phi_i = (\alpha_i + \beta_i)$, $\beta(B) = 1 - \sum_{j=1}^{q} \beta_j B^j$ and $r = \max(p, q)$. We shall make the following stationarity assumptions for y_t^2 which has an ARMA(r, q) representation.

(A.1) All zeros of the polynomial $\phi(B)$ lie outside of the unit circle.
(A.2) $\sum_{i=0}^{\infty} \psi_j^2 < \infty$ where the ψ_j's are obtained from the relation $\psi(B)\,\phi(B) = \beta(B)$ with $\psi(B) = 1 + \sum_{j=1}^{\infty} \psi_j B^j$.

The assumptions ensure that the $u_t's$ are uncorrelated with zero mean and finite variance and that the y_t^2 process is weakly stationary. In this case, the autocorrelation function of y_t^2 will be exactly the same as that for a stationary ARMA(r, q) model. For any random variable X with finite fourth moment, the kurtosis is defined by $\frac{E(X-\mu)^4}{[\mathrm{Var}(X)]^2}$. If the process $\{Z_t\}$ is normal, then the process $\{y_t\}$ defined by Eqs. (1.1) and (1.2) is called a normal GARCH(p, q) process. The kurtosis of the GARCH process is denoted by $K^{(y)}$ when it exists. In order to calculate the GARCH kurtosis in terms of ψ-weights and the ACF of the squared process, we have the following theorem given in Thavaneswaran et al. [3].

Theorem 1.1. *For the $GARCH(p, q)$ process specified by (1.1) and (1.2), under the stationarity assumptions and finite fourth moment, the kurtosis $K^{(y)}$ of the process is given by:*

(a) $K^{(y)} = \frac{E(Z_t^4)}{E(Z_t^4) - [E(Z_t^4) - 1] \sum_{j=0}^{\infty} \psi_j^2}$,

(b) (i) *The variance of the y_t^2 process is* $\gamma_0^{y^2} = \sigma_u^2 \sum_{j=0}^{\infty} \psi_j^2$,

(ii) *The k-lag autocovariance of the y_t^2 process is* $\gamma_k^{y^2} = \sigma_u^2 \sum_{j=0}^{\infty} \psi_{j+k} \psi_j$ *and for* $k \geq 1$,

(iii) *The k-lag autocorrelation is given by* $\rho_k^{y^2} = \frac{\gamma_k^{y^2}}{\gamma_0^{y^2}} = \frac{\sum_{j=0}^{\infty} \psi_{j+k} \psi_j}{\sum_{j=0}^{\infty} \psi_j^2}$.

(c) *For a normal GARCH (p, q) process* $K^{(y)} = \frac{3}{1 - 2\sum_{j=1}^{\infty} \psi_j^2}$.

The theorem can be used to identify the order of the GARCH process and identify the marginal distribution of Z_t.