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A computational comparison between two evaluation criteria in fuzzy multiobjective linear programs using possibility programming

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Abstract

In this paper, a fuzzy multiobjective linear programming model is presented. Both the objective functions and the constraints are considered fuzzy. The coefficients of the decision variables in the objective functions and in the constraints, as well as the right-hand side of the constraints are assumed to be fuzzy numbers with either trapezoidal or triangular membership functions. The possibility programming approach is utilized to transform the fuzzy model to its crisp equivalent. A comparison between the global criterion method and the distance functions method, as two evaluation criteria, is illustrated by a computational study.

Keywords: Fuzzy multiobjective linear programming; Possibility programming; Global criterion method; Distance functions method

1. Introduction

Fuzzy mathematical programming has been investigated and developed in several research studies. One of the important early contributions in fuzzy programming was given by Zimmermann [1,2]. In fuzzy multiobjective programming, Sakawa et al. [3] have presented an interactive fuzzy approach for multiobjective linear programming problems. Moreover, the fuzzy multiobjective programming was handled in the form of linear fractional models [4, 5]. Furthermore, the fuzzy multiobjective programs have been considered under randomness, whether in the case of linear programs [6,7] or that of linear fractional programs [8]. One of the main approaches in dealing with fuzzy models is the possibility theory. The basic work in possibility theory was introduced by Dubois and Prade [9]. Their work has presented the foundation of the possibility programming approach, which has been applied to fuzzy linear single-objective and multiobjective programming models [10,11] and stochastic fuzzy multiobjective programming models [6–8].

In this paper, the possibility programming approach is utilized to transform the fuzzy multiobjective linear programming model given by Negi and Lee [10] to its crisp equivalent, according to Iskander's modifications [8, 11]. The transformed crisp multiobjective linear programming model is solved using the global criterion method and the distance functions method, which are two main criteria having the same concept of evaluation. The fuzzy multiobjective linear programming model and its crisp equivalent are presented in the next section. The third section provides a computational study to compare between the two solving methods. Finally, conclusions are given in the last section.

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2. Possibility programming in fuzzy multiobjective linear programs

Consider the formulation of the fuzzy multiobjective linear programming model [10,11] as

$$\text{Maximize} \quad \sum_{j=1}^{n} \tilde{c}_{rj} x_j, \quad r = 1, 2, \ldots, p, \tag{1}$$

$$\text{subject to:} \quad \sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i, \quad i = 1, 2, \ldots, m, \tag{2}$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n, \tag{3}$$

where x_j, $j = 1, 2, \ldots, n$ are non-negative decision variables, $\tilde{c}_{rj}$ is the fuzzy coefficient of the jth decision variable in the rth objective function. $\tilde{a}_{ij}$ represents the fuzzy coefficient of the jth decision variable in the ith constraint, while $\tilde{b}_i$ is the fuzzy right-hand side in the ith constraint. Hence, $\tilde{c}_{rj}$, $\tilde{a}_{ij}$, and $\tilde{b}_i$ are considered either trapezoidal or triangular fuzzy numbers [11], i.e., $\tilde{c}_{rj} = (\underline{c}_{rj}, c_{rj1}, c_{rj2}, \overline{c}_{rj})$, $\tilde{a}_{ij} = (\underline{a}_{ij}, a_{ij1}, a_{ij2}, \overline{a}_{ij})$, and $\tilde{b}_i = (\underline{b}_i, b_{i1}, b_{i2}, \overline{b}_i)$, in the case of trapezoidal fuzzy numbers; or $\tilde{c}_{rj} = (\underline{c}_{rj}, c_{rj0}, \overline{c}_{rj})$, $\tilde{a}_{ij} = (\underline{a}_{ij}, a_{ij0}, \overline{a}_{ij})$, and $\tilde{b}_i = (\underline{b}_i, b_{i0}, \overline{b}_i)$, in the case of triangular fuzzy numbers. The improvements in applying the possibility programming approach to fuzzy linear single-objective and multiobjective programming models are utilized, whether in the case of exceedance (dominance) possibility or in the case of strict exceedance possibility [8,11]. Thus, according to trapezoidal fuzzy numbers, the equivalent crisp model for the fuzzy model (1)–(3) is given below.

2.1. *In the case of exceedance possibility*

$$\text{Maximize} \quad Z_r = \sum_{j=1}^{n} ((1-\alpha)\overline{c}_{rj} + \alpha\, c_{rj2})\, x_j, \quad r = 1, 2, \ldots, p, \tag{4}$$

$$\text{subject to:} \quad \sum_{j=1}^{n} ((1-\alpha)\underline{a}_{ij} + \alpha\, a_{ij1})\, x_j \le (1-\alpha)\overline{b}_i + \alpha\, b_{i2}, \quad i = 1, 2, \ldots, m, \tag{5}$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n, \tag{6}$$

where α is a predetermined value of the minimum required possibility, $\alpha \in (0, 1]$.

2.2. *In the case of strict exceedance possibility*

In this case, constraint set (5) in the crisp model (4)–(6) is replaced by the following constraint set:

$$\sum_{j=1}^{n} ((1-\alpha)\, a_{ij2} + \alpha\, \overline{a}_{ij})\, x_j \le (1-\alpha)\overline{b}_i + \alpha\, b_{i2}, \quad i = 1, 2, \ldots, m. \tag{7}$$

On the other hand, if $\tilde{c}_{rj}$, $\tilde{a}_{ij}$, and $\tilde{b}_i$ are triangular fuzzy numbers, then c_{rj2}, both a_{ij1} and a_{ij2}, and b_{i2} should be replaced by c_{rj0}, a_{ij0}, and b_{i0}, respectively, in the crisp model, whether in the case of exceedance possibility or in the case of strict exceedance possibility. Note that some of the fuzzy numbers may be trapezoidal while the others may be triangular, in the same fuzzy model.

In solving the crisp multiobjective linear programming problems, whether in the case of exceedance possibility or in the case of strict exceedance possibility, two evaluation criteria are used, for the purpose of comparison. Although there are several methods for solving multiobjective programming problems, two main methods, which are based on similar concept, are utilized for the set of objectives (4). The global criterion method and the distance functions method are presented, respectively, by (8) and (9) as

$$\text{Minimize } F_1 = \sum_{r=1}^{p} ((Z_{0r} - Z_r)/Z_{0r})^q, \tag{8}$$