Available online at www.sciencedirect.com
ScienceDirect
Computers and Mathematics with Applications 55 (2008) 149–161
An International Journal
computers & mathematics with applications
www.elsevier.com/locate/camwa

Modelling the frying of a non-deformable specimen by immersion in edible oil

A. Fasano*, A. Mancini

Dipartimento di Matematica "U. Dini", Università degli Studi di Firenze, viale Morgagni 67/a, 50134 Firenze, Italy

Received 8 November 2006; accepted 5 April 2007

Abstract

We present a non-trivial generalization of the one-dimensional model proposed in [A. Fasano, A. Mancini, A mathematical model for a class of frying processes, Comput. Math. Appl. (2007) (doi:10.1016/j.camwa.2006.02.046) (in press)] with several purposes. The first goal is to consider a multidimensional case (so that, for instance, the model can be applicable to French fries). We want also to provide a more realistic description of the mechanism of transport of liquid water within the sample, considering the effect of capillarity.

Keywords: Free boundary problems; Phase change; Heat and mass transfer

1. Introduction

In a previous paper [3], starting from some basic ideas proposed in [4], we have formulated and discussed a one-dimensional mathematical model for frying processes by deep immersion in hot edible oil (sketched in Fig. 2). The sample considered had to be thick enough to neglect deformations and to make the one-dimensional setting meaningful, at least in a central region.

Here, we keep the assumption of no deformation, but we make one more step towards the description of the real process by considering the multidimensional case. As we shall see, this generalization does not consist in just replacing scalars by vectors, owing in particular to the presence of unilateral constraints which may produce complex structures on the interface of total vaporization. We will also refine some aspects (e.g. the evolution of permeability to vapour) which may have some practical relevance. In its present formulation, the model will be applicable e.g. to samples showing negligible deformations, but with geometry far from 1-D, as with French fries. As in [3], we have to write:

1. the governing equations for the main three regions:
   - (Ω_{sat}) the core (water saturated below the boiling point),
   - (Ω_{mix}) the mixed region (with vapour in equilibrium with liquid water),
   - (Ω_{vap}) the fully vaporized region.

   The crust is a thin outer layer whose properties will be incorporated in the boundary conditions.

* Corresponding author. Fax: +39 055 4573898.
E-mail address: fasano@math.unifi.it (A. Fasano).

0898-1221/$ - see front matter
doi:10.1016/j.camwa.2007.04.010

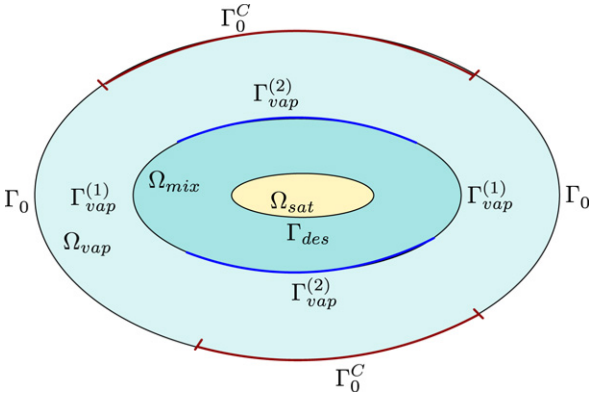

Fig. 1. Schematic representation of domains and boundaries: water saturated core (Ω_{sat}), mixed region (Ω_{mix}), vapour region (Ω_{vap}); $\Omega_{\text{sat}}/\Omega_{\text{mix}}$ desaturation interface (Γ_{des}), $\Omega_{\text{mix}}/\Omega_{\text{vap}}$ total vaporization interface ($\Gamma_{\text{vap}}^{(1)}$ unconstrained, $\Gamma_{\text{vap}}^{(2)}$ constrained), outer boundary (Γ_0 without crust, Γ_0^C with crust).

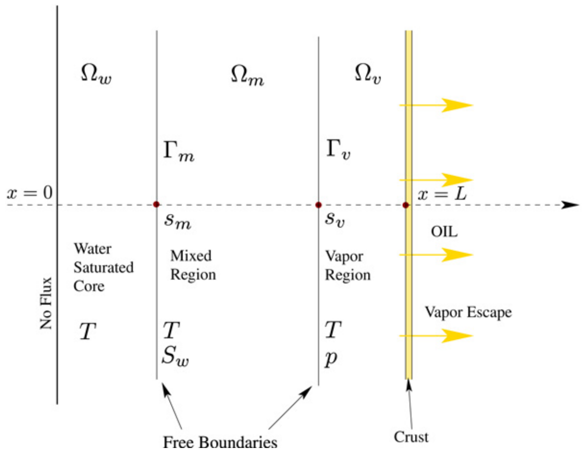

Fig. 2. Schematic representation of the problem in 1D.

2. The interface conditions expressing mass and enthalpy conservation.
3. The conditions on the outer boundary.

While in [3] the sample was treated as a porous medium with constant porosity and with permeability to vapour jumping across the $\Omega_{\text{mix}}/\Omega_{\text{vap}}$ interface, here we introduce a more realistic kinetics for the evolution of this quantity and, above all, we will model liquid water transport on the basis of Darcy's law and of capillarity effects, differently from the Fickian type law taken in [1] and [2].

The model is formulated in Sections 1–4. Some simplifications can be obtained passing to the non-dimensional formulation, in which many different characteristic times come into play (see Section 5).

2. Formulation of the mathematical model: The governing differential equations

We refer to the generic situation sketched in Fig. 1.

Ω_{sat} — The water saturated region (water saturation $S_w \equiv 1$)

Heat balance

The heat capacity per unit volume and the conductivity are expressed by

$$C_{\text{sat}} = \varepsilon \rho_w c_w + (1 - \varepsilon)\rho_s c_s, \quad k_{\text{sat}} = \varepsilon k_w + (1 - \varepsilon)k_s, \tag{2.1}$$