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Fractional-order $PI^{\lambda}D^{\mu}$ controller design

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ABSTRACT

This paper introduces a new design method of fractional-order proportional–derivative (FOPD) and fractional-order proportional–integral–derivative (FOPID) controllers. A biquadratic approximation of a fractional-order differential operator is used to introduce a new structure of finite-order FOPID controllers. Using the new FOPD controllers, the controlled systems can achieve the desired phase margins without migrating the gain crossover frequency of the uncontrolled system. This may not be guaranteed when using FOPID controllers. The proposed FOPID controller has a smaller number of parameters to tune than its existing counterparts. A systematic design procedure is identified in terms of the desired phase and the gain margins of the controlled systems. The viability of the design methods is verified using a simple numerical example.

1. Introduction

Fractional calculus is a generalization of the classical integer-order calculus that includes integro-differential operators of fractional orders. Fractional-order calculus has gained extensive attention lately since many systems in chemistry, physics, and in engineering manifest a memory effect and they are best described by fractional-order dynamics [1–3]. Due to the increase in system applications, considerable attention has been given to exact and numerical solutions of fractional-order integro-differential equations [4]. The existence and uniqueness of solutions of linear and nonlinear fractional-order integro-differential equations were discussed by Babakhani and Baleanu [5].

The performance of fractional-order systems can be manipulated by implementing integer or fractional-order control algorithms. In many applications, it has been demonstrated that fractional-order controllers have superseded their integer-order counterparts [6–8].

The proportional–integral–derivative (PID) controller is one such controller that has been successfully used in industrial applications for several decades. The popularity of the PID controller lies in the simplicity of the design procedures and in the effectiveness of its system performance [9]. A fractional-order PID controller (FOPID), on the other hand, was introduced in [10,3,11]; it is a generalization of the conventional integer-order PID controller. It is denoted by $PI^{\lambda}D^{\mu}$, where λ and μ are two additional parameters to the integral and the derivative components of the conventional PID controller, thus increasing the complexity of tuning these parameters.

Several attempts to find an optimum setting for the five different parameters of the fractional $PI^{\lambda}D^{\mu}$ controller, in order to achieve predefined design requirements, are presented in [7,12]. The tuning rules of Ziegler–Nichols for an FOPID controller were reported in [13]. New tuning algorithms for FOPID controllers are recently presented in [6,14]. The validity of an optimum FOPID controller tuned by a particle swarm was also demonstrated by Karimi et al. [15] to control the automatic voltage regulator (AVR) of a power system.

The existing design and tuning algorithms of FOPID controllers were demonstrated via numerical simulations using MATLAB/SIMULINK toolboxes [16]; however, hardware realization of FOPID controllers is more challenging [8,17,18]. The proposed method intends to look for new realizable forms and new tuning rules for FOPID controllers.

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The rest of the paper is organized as follows. Section 2 describes a fundamental overview of fractional-order systems. Section 3 provides a new fractional-order lead controller design, which is a rudimental to a new FOPID controller. Section 4 introduces a new design method of FOPID controllers. The main ideas of the design are described by numerical examples embedded when necessary. Finally, Section 5 summarizes the main points of this work.

2. Fractional-order systems overview

Fractional-order systems, which are based on fractional-order calculus [19,20], are a generalization of dynamical systems that exhibit non-Newtonian behavior [21]. The integer-order dynamics describe special and smaller class of fractional-order systems. Consequently, fractional-order controllers, as demonstrated by many researchers, such as Podlubny et al. [11] and El-Khazali et al. [22], outperformed their integer-order counterparts.

Fractional-order systems are described by an n-term non-homogeneous fractional-order differential equation (FDE) of the form

$$a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \cdots a_1 D^{\alpha_1} y(t) + a_0 D^{\alpha_0} y(t) = b_m D^{\beta_m} u(t) + b_1 D^{\beta_1} u(t) + b_0 D^{\beta_0} u(t), \quad (1)$$

where $D^{\alpha} = {}_0D_t^{\alpha}$ is the Caputo fractional derivative of order α_k; $k = 1, 2, \ldots, n$, and where β_l $(l = 0, 1, 2, \ldots, m)$ are arbitrary constants.

One may assume, without loss of generality, that $0 = \alpha_0 < \alpha_1 < \alpha_2 \cdots < \alpha_n$, and $0 = \beta_0 < \beta_1 < \beta_2 \cdots < \beta_m$. The Laplace transform of the fractional-order derivative, $D^{\alpha} y(t)$; $0 < \alpha \le 1$, is given by Podlubny [3]:

$$\int_0^{\infty} D^{\alpha} y(t) e^{-st} dt = s^{\alpha} Y(s) - \sum_{k=0}^{N} s^{\alpha-k-1} y^{(k)}(0), \quad (2)$$

while the fractional-order integral of y is denoted by ${}_0I_t^{-\alpha} y(t)$, and the sum in the right-hand side in (2) is omitted [12]. Therefore, for zero initial conditions, the transfer function of the fractional-order system described in (1) is given by

$$G_p(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots a_1 s^{\alpha_1} + a_0 s^{\alpha_0}}. \quad (3)$$

The goal of this work is to design an FOPID controller of the form

$$G_c(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}. \quad (4)$$

The complexity of this controller is evident due to the increase in the number of control parameters. There are five different parameters (K_p, K_i, K_d, λ, and μ) that have to be tuned, which increases the flexibility of achieving preset design requirements such as steady-state errors, phase and gain margins, and robustness.

The challenge of this work is to develop a realizable FOPID controller that exhibits a robust performance with a smaller number of parameters, yet achieving the same design requirements. The key point is to look for acceptable and realizable approximations to the differential operators, s^{λ}, and s^{μ}. The next section introduces a new design technique of a fractional-order PD^{μ} controller that paves the way for a new design technique of an FOPID controller.

3. Fractional-order lead controller design

The proportional–derivative controller represents a class of a typical lead controller. Lead compensators are usually cascaded with uncompensated plants to add a leading phase to stabilize and to reshape the plant's frequency response. The conventional lead compensator adds additional phase and gain to the uncontrolled system by carefully selecting its poles and zeros in the complex plane [23]. Usually, the design process does not succeed at the first attempt and it requires a trial and error process to achieve the design requirements.

To alleviate the problem of selecting the poles and zeros of PD^{μ} controllers, a biquadratic structure of the fractional-order differential operator can be used to approximate the performance of PD^{μ} controllers within an operating frequency band [18]. Once the first biquadratic structure is designed, higher-order PD^{μ} controllers are obtained by cascading several modules. Thus, it can be considered as a modular controller design, in which several modules (each of biquadratic transfer function) are automatically cascaded and normalized at different cut-off frequencies to improve the robustness of the controlled system by widening the flatness of its phase response.

Now, consider the approximation of a fractional operator, s^{μ}, by the following biquadratic approximation [24]:

$$s^{\mu} \approx \frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \equiv T(s) \equiv \frac{N(s)}{D(s)}, \quad (5)$$

where a_0, a_1, and a_2 are real constants.

Selecting a_0, a_1, and a_2 properly can approximate a fractional differential (integral) operator within a band-limited frequency spectrum, i.e., it can be used to design a PD^{μ} (or a proportional–integral PI^{μ} fractional-order) controller [18].