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Sustaining stable dynamics of a fractional-order chaotic financial system by parameter switching

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ABSTRACT

In this paper, a simple parameter switching (PS) methodology is proposed for sustaining the stable dynamics of a fractional-order chaotic financial system. This is achieved by switching a controllable parameter of the system, within a chosen set of values and for relatively short periods of time. The effectiveness of the method is confirmed from a computer-aided approach, and its applications to chaos control and anti-control are demonstrated. In order to obtain a numerical solution of the fractional-order financial system, a variant of the Grünwald–Letnikov scheme is used. Extensive simulation results show that the resulting chaotic attractor well represents a numerical approximation of the underlying chaotic attractor, which is obtained by applying the average of the switched values. Moreover, it is illustrated that this approach is also applicable to the integer-order financial system.

1. Introduction

Today, many economists still focus on linear dynamics (e.g., using the Hartman–Grobman theorem), thinking of that nonlinear dynamics are intractable although the economic world is by nature nonlinear. Nevertheless, the intrinsic relation between chaos theory and finance theory has been widely explored since the pioneering work of Smale in 1953 [1]. As a result, financial systems are commonly modeled by continuous-time chaotic systems such as the forced Van-der-Pol model [2], Behrens–Feichtinger model [3], Cournot–Puu model [4], IS–ML model [5], and so on (see also [6–9] and references therein). In addition, many recent studies on economics have demonstrated the adverse effect of chaotic dynamics on economic systems.

Due to the instability of a periodic solution, bifurcation, or other typical phenomena which could appear in chaotic economic systems, some measures and actions are required. Many researchers suggested applying chaos control in financial systems in order to improve their performances such as preserving stability. Indeed, controlling a chaotic market model may lead to economic efficiency. Therefore, interest in suppressing chaos in economic models has been raised from the scientific community [10–14].

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In this paper, the study is devoted to the chaotic financial system introduced in [15], which is originally of integer-order, but has lately being extended to fractional-order in [16]. The system is described by the following differential equations:

$$\begin{aligned}\frac{dx_1^{q_1}}{dt^{q_1}} &= x_3 + (x_2 - p)x_1,\\ \frac{dx_2^{q_2}}{dt^{q_2}} &= 1 - bx_2 - x_1^2,\\ \frac{dx_3^{q_3}}{dt^{q_3}} &= -x_1 - cx_3,\end{aligned} \tag{1}$$

where p, b and c are nonnegative coefficients with physical meanings and significance clearly explained in [17]; $q = (q_1, q_2, q_3)^T$ represents the fractional order of the derivatives, in which $q_i \in (0, 1]$, with $q_i = 1,\ i = 1, 2, 3$, representing the integer-order case.

For the integer-order financial system (1), i.e., with $q_i = 1,\ i = 1, 2, 3$, its local topological structure and bifurcation have been studied in detail (see [15,17]). For its fractional-order version, the nonlinear dynamics have also been analyzed in [16,18]. Furthermore, this financial system model has been investigated regarding chaos control and synchronization in [14,19].

In this paper, we show numerically that any stable attractor of the financial system (1) can be approximated by switching p within a set of chosen values in deterministic and relatively small time intervals. Compared to other methods, such as OGY-like schemes, where unstable periodic orbits are "forced" to become stable, here one obtains whatever stable attractor that is desirable.

The system (1) can be reformulated as the following general initial value problem (IVP):

$$\frac{d^q}{dt^q}x(t) = f(x(t)) = g(x(t)) + pAx(t), \qquad x(0) = x_0, \quad t \in I = [0, T], \tag{2}$$

where $x : I \to \mathbb{R}^3$, $g : \mathbb{R}^3 \to \mathbb{R}^3$ is a continuous nonlinear function, A is a 3×3 real constant matrix, and p is a tunable real parameter to be used for control by switching its values later.

The IVP (2) is useful for describing a large class of well-known dynamical systems of integer-order or fractional-order, for example the Lorenz, Rössler, Chen, Chua systems, to name just a few.

Referring to (1), one has

$$g(x) = \begin{pmatrix} x_3 + x_1x_2 \\ 1 - bx_2 - x_1^2 \\ -x_1 - cx_3 \end{pmatrix}, \qquad A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

When $q = 1$, system (2) corresponds to a classical first-order IVP, which can be numerically solved by standard methods, such as Runge–Kutta. On the other hand, for $q \in (0, 1)$, system (2) becomes an IVP of fractional-order, presented as fractional differential equation (FDE). In this case, we consider the fractional derivative operator d^q/dt^q as being Caputo's derivative with starting point $t_0 = 0$, namely,

$$\frac{d^q}{dt^q}x(t) = \frac{1}{\Gamma(1-q)} \int_0^t \big(t-s\big)^{-q} x'(s)ds, \tag{3}$$

where Γ is the Euler gamma function (for basic knowledge on fractional calculus, one may refer to [20–25]). The use of Caputo's approach allows coupling the FDE with initial conditions in a classical form as in (2) and, unlike the Riemann–Liouville (RL) definition

$${}^{\mathrm{RL}}D_0^q x(t) = \frac{1}{\Gamma(1-q)} \frac{d}{dt} \int_0^t \big(t-s\big)^{-q} x(s)ds,$$

it avoids the expression of initial conditions with fractional derivatives. However, under the assumption that x is absolutely continuous, the Caputo and RL definitions are related by a relationship involving the initial condition

$$\frac{d^q}{dt^q}x(t) = {}^{\mathrm{RL}}D_0^q\big(x(t) - x(0)\big) \tag{4}$$

(one can refer to [20] for more insights on this topic and for the extension of the above definitions and relationship to the case $q > 1$). Another approach to introduce derivatives of non-integer order is the Grünwald–Letnikov (GL) operator

$${}^{\mathrm{GL}}D_0^q x(t) = \lim_{N\to\infty} h_N^{-q} \sum_{k=0}^{N} \omega_k^{(q)} x(t - kh_N), \quad h_N = t/N, \tag{5}$$