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A new approach for solving a system of fractional partial differential equations

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A R T I C L E I N F O

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A B S T R A C T

In this paper we propose a new method for solving systems of linear and nonlinear fractional partial differential equations. This method is a combination of the Laplace transform method and the Iterative method and here after called the Iterative Laplace transform method. This method gives solutions without any discretization and restrictive assumptions. The method is free from round-off errors and as a result the numerical computations are reduced. The fractional derivative is described in the Caputo sense. Finally, numerical examples are presented to illustrate the preciseness and effectiveness of the new technique.

1. Introduction

Many problems of mathematical physics and engineering such as polymer physics, viscoelastic materials, viscous damping and seismic analysis [1–4] have been successfully modeled in recent years by fractional differential equations (FDEs). So it is very important to find efficient methods for solving FDEs. Various researchers have introduced new methods in the literature. These methods include the Adomian decomposition method (ADM) [5,6], homotopy analysis method (HAM) [7,8], homotopy perturbation method (HPM) [9,10], the variational iteration method (VIM) [6,11,12] and the Laplace decomposition method [13–15].

Recently, a new iterative method was presented by Daftardar-Gejji and Jafari [16,17]. This technique solves many types of nonlinear equations such as ordinary and partial differential equations of integer and fractional order. Jafari et al. [18] applied this method to obtain the solution of linear/nonlinear diffusion and wave fractional equations. Daftardar-Gejji and Bhalekar used it for solving fractional boundary value problem and evolution equations [19,20].

In this paper, we introduce a new method, which we call the Iterative Laplace transform method (ILTM). The suggested ILTM provides the solution in a rapid convergent series which may lead us to the solution in a closed form. This method combines the two powerful methods, namely, the Laplace transform method and the Iterative method, for obtaining the exact solution for the system of fractional partial differential equations. It is worth mentioning that the ILTM is applied without any discretization or restrictive assumptions or transformations and it is free from round-off errors. Also this method provides an analytical solution by using the initial conditions only, unlike the variables separation method, which requires initial and boundary conditions. The boundary conditions can be used to justify the obtained results. The proposed method

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works efficiently and the results so far are very encouraging and reliable. In this paper we employ the ILTM in solving systems of nonlinear fractional partial differential equations. Several examples are given to verify the reliability and efficiency of the ILTM. The results are then compared with those obtained by other existing methods.

2. Basic definition

In this section, we recall some basic definitions and results dealing with the fractional calculus [2–4] and Laplace transform which are later used in this paper.

Definition 1. A real function $f(t),\ t > 0$ is said to be in the space $C_\alpha, \alpha \in \Re$ if there exists a real number $p(> \alpha)$, such that $f(t) = t^p f_1(t)$ where $f_1 \in C[0, \infty]$. Clearly $C_\alpha \subset C_\beta$ if $\beta \le \alpha$.

Definition 2. A function $f(t),\ t > 0$ is said to be in the space C_α^m, $m \in N \bigcup \{0\}$, if $f^{(m)} \in C_\alpha$.

Definition 3 ([2]). The left sided Riemann–Liouville fractional integral of order $\mu \ge 0$, of a function $f \in C_\alpha,\ \alpha \ge -1$ is defined as

$$I^\mu f(t) = \begin{cases} \dfrac{1}{\Gamma(\mu)} \displaystyle\int_0^t \frac{f(\tau)}{(t-\tau)^{1-\mu}} d\tau, & \mu > 0, t > 0, \\ f(t), & \mu = 0. \end{cases} \tag{1}$$

Definition 4 ([2]). The left sided Caputo fractional derivative of $f, f \in C_{-1}^m,\ m \in N \cup \{0\}$, is defined as

$$D^\mu f(t) = \frac{\partial^\mu f(t)}{\partial t^\mu} = \begin{cases} I^{m-\mu}\left[\dfrac{\partial^m f(t)}{\partial t^m}\right], & m-1 < \mu < m, m \in N, \\ \dfrac{\partial^m f(t)}{\partial t^m}, & \mu = m. \end{cases} \tag{2}$$

Note that

(i) $I_t^\mu f(x,t) = \frac{1}{\Gamma(\mu)} \int_0^t \frac{f(x,t)}{(t-s)^{1-\mu}},\ \mu > 0,\ t > 0,$

(ii) $D_t^\mu f(x,t) = I_t^{m-\mu} \frac{\partial^m f(x,t)}{\partial t^m},\ m-1 < \mu \le m.$

Definition 5. The Mittag-Leffler function $E_\alpha(z)$ with $\alpha > 0$ is defined by the following series representation, valid in the whole complex plane:

$$E_\mu(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\mu n + 1)}, \quad \alpha > 0,\ z \in \mathbb{C}.$$

Definition 6. The Laplace transform of $f(t)$ is defined by

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t) dt. \tag{3}$$

Definition 7. The Laplace transform $L[f(t)]$ of the Riemann–Liouville fractional integral is defined as

$$L\{I^\mu f(t)\} = s^{-\mu} F(s). \tag{4}$$

Definition 8. The Laplace transform $L[f(t)]$, of the Caputo fractional derivative is defined as

$$L\{D^\mu f(t)\} = s^\mu F(s) - \sum_{k=0}^{n-1} s^{(\mu-k-1)} f^{(k)}(0), \quad n-1 < \mu \le n. \tag{5}$$

3. Iterative Laplace transform method and system of fractional partial differential equations

To illustrate the basic idea of this method, we consider the following system of fractional partial differential equations (FPDEs) with the initial conditions of the form:

$$D_t^{\alpha_i} u_i(\bar{x}, t) = A_i(u_1(\bar{x}, t), \ldots, u_n(\bar{x}, t)), \quad m_i - 1 < \alpha_i \le m_i, \quad i = 1, 2, \ldots, n, \tag{6}$$

$$\frac{\partial^{(k_i)} u_i(\bar{x}, 0)}{\partial t^{k_i}} = h_{ik_i}(\bar{x}), \quad k_i = 0, 1, \ldots, m_i - 1,\ m_i \in N, \tag{7}$$