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Analytical VaR for international portfolios with common jumps

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ABSTRACT

International portfolios which are composed of domestic assets and foreign assets are popular investment tools for financial institutions in highly integrated global financial markets. However, the focus of past studies had been on either domestic assets or foreign assets, but not both in the same context. They paid no attention to the studies of controlling the market risk of the international portfolios in the risk management literature. In contrast to the existing literature in portfolios, this paper considers not only domestic assets but also foreign assets, and provides an analytical value-at-risk (VaR) with common jump risk and exchange rate risk to manage market risk of international portfolios with exchange rate risk and common jumps over the subprime mortgage crisis. In general, the analytical solution can be used to accurately calculate VaRs by the backtesting criterion in terms of in-sample and out-of-sample fitting for an international portfolio with common jumps.

1. Introduction

Nowadays, the investment in foreign currency assets circulates rapidly around the world. In Taiwan, the official monthly statistic reports offered by the Central Bank of Taiwan illustrate that the average percentage of investment in foreign assets relative to domestic assets has been approximately 46% at domestic commercial banks over the past ten years. In Japan, the ratio is at least 5%, and in Korea it is around 9%. On average, the percentage of overall portfolio allocation to foreign assets is around 20% at Asian banks, and the percentage is growing. Thus, controlling the market risk of portfolios composed of domestic assets and foreign assets is an increasing concern for financial institutions.

The VaR approach is a popular tool to manage market risk, which is defined as the maximum loss over a fixed target horizon with a given probability. Using the VaR measure, Hofmann and Platen [1] consider the market risk of a large diversified portfolio in which the dynamic process of asset returns is distributed in normal diffusion. Equally, the asset price follows a lognormal distribution. However, substantial evidence exists in the empirical financial economic literature of the existence of jumps in equity returns and foreign exchange rates such as [2–4]. Therefore, the lognormal assumption is, in actuality, contrary to real life. Daily changes in many variables, especially in exchange rates, illustrate significant positive kurtosis. This means that the probability distributions of asset returns have fat tails or discontinuity. Literature related to these studies has been presented by Stock and Watson [5], Hull and White [6], Hansen [7], and Consigli [8]. Besides them, Shang et al. [9] employ a jump–diffusion process to price catastrophe mortality bonds; Liu et al. [10] consider a class of stochastic optimal parameter selection problems described by linear stochastic differential equations with jumps to show that the constrained stochastic impulsive optimal parameter selection problem is equivalent to a deterministic impulsive optimal parameter; Ma and Zhao [11] and Tin et al. [12] apply a jump–diffusion process to a simulation analysis of nearest-neighbour rule under stochastic demand and a web reliability ranking system. Alternatively, Gibson [13] demonstrates that event risk poses large jumps to fat tails in market prices, and incorporates event risk into VaR for a portfolio. Differing from

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the assumption held by Hofmann and Platen [1] and Guan et al. [14], Gibson considers jump–diffusion asset returns to model large diversified portfolios. As stated above, the literature focuses on the portfolios only valued in one currency. However, it is a common phenomenon for institutional and individual investors to invest in the portfolios which include a number of domestic-valued assets and foreign-valued assets in highly integrated global financial markets, called international portfolios. Therefore, exchange rate risk should be considered in highly international investment.

This paper aims to present an analytical VaR formula for international portfolios. Using the framework provided by Merton [15], we employ return jumps at Poisson arrivals to avoid the assumption of normality of asset returns. Also, the Brownian motions of between-jump returns are correlated. In general, the model solution is more accurate than that of the Monte Carlo simulation techniques which are often adopted in fat-tail distributions in terms of the system infrastructure and computation time. In addition, this model can be also applied to large portfolios. Compared with that of Hofmann and Platen [1] and Guan et al. [14], the proposed model considers not only jumps but also exchange rate risk. It is more suitable to fit to real situations in highly integrated global financial markets.

The rest of this paper is organized as follows. The next section outlines the model, and an analytic formula of the value at risk is derived. In the Section 3, we first employ an international portfolio including domestic assets and foreign assets to estimate model parameters. Then, the one-day VaRs at 99% significance level for the international portfolio are calculated, and a comparative static analysis on the risk capital is provided. Using the usual backtesting criterion, Section 4 inspects the model accuracy in terms of in-sample and out-of-sample fitting over the subprime mortgage crisis of August 2007. The samples in this study span from January 1, 2004 to November 27, 2009, or 1367 daily log returns of a line of domestic assets and foreign assets. The last section provides conclusions.

2. Model formulation

First, this paper assumes that (i) a value of an international portfolio is made up of the value of n_d kinds of domestic assets with $m_{i,t}$ shares and n_f classes of foreign assets with $g_{i,t}$ shares for each $i \in 1, 2, \ldots, n$; (ii) the capital market is a complete market with no transaction cost or tax; (iii) there exists a riskless interest rate for lenders and borrowers; (iv) the dynamic processes of domestic asset returns, foreign asset returns and exchange rate returns follow Poisson jump–diffusion over the interval of interest; (v) exchange rates are quoted at the price of one unit of the foreign currencies in domestic dollars, and (vi) investment strategies do not vary over an investment horizon. The dynamic processes of asset price and exchange rates are demonstrated as follows, respectively.

$$\frac{\mathrm{d}A_{d_i,t}}{A_{d_i,t}} = (\mu_{d_i} - \lambda \upsilon)\mathrm{d}t + \sigma_{d_i}\mathrm{d}W_{1,t} + (\pi - 1)\mathrm{d}Y_t, \tag{1}$$

$$\frac{\mathrm{d}A_{f_i,t}}{A_{f_i,t}} = (\mu_{f_i} - \lambda \upsilon)\mathrm{d}t + \sigma_{f_i}\mathrm{d}W_{2,t} + (\pi - 1)\mathrm{d}Y_t, \tag{2}$$

$$\frac{\mathrm{d}e_{i,t}}{e_{i,t}} = (\mu_{e_i} - \lambda \upsilon)\mathrm{d}t + \sigma_{e_i}\mathrm{d}W_{3,t} + (\pi - 1)\mathrm{d}Y_t, \tag{3}$$

where μ_{d_i}, μ_{f_i}, and μ_{e_i} denote the constant drift rates of domestic asset returns, foreign asset returns and exchange rate returns for each $i \in 1, 2, \ldots, n$, respectively; σ_{d_i}, σ_{f_i}, and $\sigma_{e_i,t}$ stand for the constant volatilities of domestic asset returns, foreign asset returns and exchange rate returns for each $i \in 1, 2, \ldots, n$, respectively. The $W_{j,t}$ are one dimensional standard Brownian motions under the original probability measure, P for all $j = 1, 2, 3$. Also, the correlation coefficients among the three Brownian motions are defined as $\mathrm{corr}(\mathrm{d}W_{1,t}, \mathrm{d}W_{2,t}) = \rho_{1,2}$, $\mathrm{corr}(\mathrm{d}W_{2,t}, \mathrm{d}W_{3,t}) = \rho_{2,3}$, and $\mathrm{corr}(\mathrm{d}W_{1,t}, \mathrm{d}W_{3,t}) = \rho_{1,3}$.[1] Then, Y_t is an independent Poisson process with the intensity λ at time t; $\mathrm{d}Y_t$ is independent of $\mathrm{d}W_{j,t}$ for all $j = 1, 2, 3$. The υ represents $E[\pi - 1]$ where $\pi - 1$ is the random variable percentage in domestic assets or exchange rates resulting from a jump, and $E(.)$ is the symbol of the expectation operator over the random variable Y_t. Assume that the nature logarithm of π, which is the jump amplitude if Poisson events occur, follows normal distributions with the mean u_π and variance σ_π^2. That is also denoted as $\ln \pi \sim N(u_\pi, \sigma_\pi^2)$, and $\upsilon = E[\pi - 1] = \exp\left[u_\pi + \frac{1}{2}\sigma_\pi^2\right] - 1$.

Now, consider the potential daily loss exposure to long trading positions. Typically, the VaR is a specific left-hand critical value of a potential loss distribution. Given conventions, one can define the daily losses valued in domestic dollars relative to the end-of-period expected asset value (relative VaR) and the initial asset value (absolute VaR), denoted by VaR(*mean*) and VaR(0) as follows, respectively:

$$\begin{aligned} \mathrm{VaR}(mean) &\equiv V_\alpha - E_t(V_T), \\ \mathrm{VaR}(0) &\equiv V_\alpha - V_0, \end{aligned} \tag{4}$$

[1] For simplicity, we assume that the dependence structure between exchange rates and equity returns is linear. However, there are some drawbacks. First, it is not invariant to transformations of the original variables. Second, conditional correlations are not accounted for. Third, the proposed method cannot be used in the case of portfolios that include assets with non-linear payoffs.