Computers and Mathematics with Applications 62 (2011) 3103–3110

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Discontinuity of the trapezoidal fuzzy number-valued operators preserving core

Adrian I. Ban [a,*], Lucian C. Coroianu [a,b]

[a] Department of Mathematics and Informatics, University of Oradea, Universităţii 1, 410087 Oradea, Romania

[b] Department of Mathematics, Babes-Bolyai University, Mihail Kogălniceanu 1, 400084 Cluj-Napoca, Romania

ARTICLE INFO

Article history:
Received 23 May 2010
Received in revised form 5 August 2011
Accepted 8 August 2011

Keywords:
Fuzzy number
Trapezoidal fuzzy number
Weighted trapezoidal approximation
Discontinuity

ABSTRACT

We prove that any trapezoidal fuzzy number-valued operator preserving core is discontinuous with respect to any weighted metric on the space of fuzzy numbers. As an application, we obtain the discontinuity of the weighted trapezoidal approximation operator preserving core.

1. Introduction

The continuity is considered between the essential properties of an approximation operator. In applications, where it is sometimes indicated as robustness, the criterion of continuity is of extreme importance too. In the present paper we prove that any trapezoidal fuzzy number-valued operator preserving core is discontinuous with respect to any weighted metric and each fuzzy number with 1-cut set as a proper interval is a point of discontinuity. As an immediate consequence, the result of continuity of the trapezoidal approximation operator preserving core in [1] is not valid. Nevertheless, this operator has a good property proved in the final part of the paper. These results are important because continuity is between the criteria that a trapezoidal (triangular, parametric) approximation operator should possesses (see [2]). In the past years many researchers have focused on finding such approximations of fuzzy numbers, with respect to average Euclidean or weighted metrics, with or without additional conditions (see [1–11]). The aim is the simplification of calculus for easy interpretation and implementation, under conditions of preservation of information, in applications related with multiple areas where the fuzzy numbers are used to represent uncertain and incomplete information: decision making, linguistic controllers, biotechnological systems, expert systems, data mining, pattern recognition, etc.

2. Preliminaries

We consider the following well-known description of a fuzzy number A:

$$A(x)=\begin{cases}0, & \text{if } x\le a_1\\ l_A(x), & \text{if } a_1\le x\le a_2\\ 1 & \text{if } a_2\le x\le a_3\\ r_A(x), & \text{if } a_3\le x\le a_4\\ 0, & \text{if } a_4\le x,\end{cases} \tag{1}$$

* Corresponding author.
E-mail addresses: aiban@uoradea.ro (A.I. Ban), lcoroianu@uoradea.ro (L.C. Coroianu).

0898-1221/$ – see front matter
doi:10.1016/j.camwa.2011.08.023

where $a_1, a_2, a_3, a_4 \in \mathbb{R}$, $l_A : [a_1, a_2] \longrightarrow [0, 1]$ is a nondecreasing upper semicontinuous function, $l_A(a_1) = 0$, $l_A(a_2) = 1$, called the left side of A and $r_A : [a_3, a_4] \longrightarrow [0, 1]$ is a nonincreasing upper semicontinuous function, $r_A(a_3) = 1$, $r_A(a_4) = 0$, called the right side of A. The α-cut, $\alpha \in (0, 1]$, of a fuzzy number A is a crisp set defined as

$$A_\alpha = \{x \in \mathbb{R} : A(x) \geq \alpha\}.$$

The support or 0-cut A_0 of A is defined as

$$A_0 = \mathrm{cl}\{x \in \mathbb{R} : A(x) > 0\},$$

where cl is the closure operator. Every α-cut, $\alpha \in [0, 1]$, of A is a closed interval

$$A_\alpha = [A_L(\alpha), A_U(\alpha)],$$

where

$$A_L(\alpha) = \inf\{x \in \mathbb{R} : A(x) \geq \alpha\},$$
$$A_U(\alpha) = \sup\{x \in \mathbb{R} : A(x) \geq \alpha\},$$

for any $\alpha \in (0, 1]$. The core of A is defined as

$$\mathrm{core}(A) = A_1 = [A_L(1), A_U(1)].$$

If the sides of the fuzzy number A are strictly monotone then one can see easily that A_L and A_U are inverse functions of l_A and r_A, respectively. We denote by $F(\mathbb{R})$ the set of all fuzzy numbers.

The average Euclidean metric $d_{1,1}$ on $F(\mathbb{R})$ is defined by [12]

$$d_{1,1}^2(A, B) = \int_0^1 (A_L(\alpha) - B_L(\alpha))^2 \mathrm{d}\alpha + \int_0^1 (A_U(\alpha) - B_U(\alpha))^2 \mathrm{d}\alpha. \quad (2)$$

A weighted metric $d_{f,g}$ on $F(\mathbb{R})$, which generalizes the above metric, is defined by (see [10])

$$d_{f,g}^2(A, B) = \int_0^1 f(\alpha)(A_L(\alpha) - B_L(\alpha))^2 \mathrm{d}\alpha + \int_0^1 g(\alpha)(A_U(\alpha) - B_U(\alpha))^2 \mathrm{d}\alpha, \quad (3)$$

where $f, g : [0, 1] \to \mathbb{R}$ are called weighted functions, that is they are integrable, nonnegative, nondecreasing and satisfy $\int_0^1 f(\alpha)\mathrm{d}\alpha > 0$ and $\int_0^1 g(\alpha)\mathrm{d}\alpha > 0$. The property of monotonicity of functions f and g means that the higher the cut level is, the more important its weight is in determining the distance of fuzzy numbers A and B. Often $f(\alpha) = g(\alpha) = 1$, for every $\alpha \in [0, 1]$, and the average Euclidean metric is obtained or $f(\alpha) = g(\alpha) = \alpha$, for every $\alpha \in [0, 1]$. Sometimes the additional conditions $f(0) = g(0) = 0$ and $\int_0^1 f(\alpha)\mathrm{d}\alpha = \int_0^1 g(\alpha)\mathrm{d}\alpha = \frac{1}{2}$ are imposed to weighted functions (see [11]).

Fuzzy numbers with simple membership functions are preferred in practice. The most often used fuzzy numbers are so-called trapezoidal fuzzy numbers. A trapezoidal fuzzy number T, $T_\alpha = [T_L(\alpha), T_U(\alpha)]$, $\alpha \in [0, 1]$, is given by

$$T_L(\alpha) = t_1 + (t_2 - t_1)\alpha$$

and

$$T_U(\alpha) = t_4 - (t_4 - t_3)\alpha,$$

with $t_1 \leq t_2 \leq t_3 \leq t_4$. We denote

$$T = (t_1, t_2, t_3, t_4)$$

a trapezoidal fuzzy number and by $F^T(\mathbb{R})$ the set of all trapezoidal fuzzy numbers.

A trapezoidal fuzzy number-valued operator $T : F(\mathbb{R}) \to F^T(\mathbb{R})$ is called (see [2]) continuous with respect to metric $d : F(\mathbb{R}) \times F(\mathbb{R}) \to [0, +\infty)$ if, for any $A, B \in F(\mathbb{R})$, we have

$$\forall \varepsilon > 0, \quad \exists \delta > 0 : d(A, B) < \delta \Longrightarrow d(T(A), T(B)) < \varepsilon.$$

3. Auxiliary results

To prove the main results of the paper, first we need some auxiliary results as follows.

Lemma 1. *Let $f : [0, 1] \to \mathbb{R}$ be a weighted function. Then*

$$\left(\int_0^1 f(\alpha)\alpha(1-\alpha)\mathrm{d}\alpha\right)^2 < \int_0^1 f(\alpha)\alpha^2 \mathrm{d}\alpha \int_0^1 f(\alpha)(1-\alpha)^2 \mathrm{d}\alpha.$$