Computers and Mathematics with Applications 62 (2011) 3241–3250

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Model order reduction methods for coupled systems in the time domain using Laguerre polynomials☆

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ARTICLE INFO

Article history:
Received 5 January 2011
Received in revised form 8 August 2011
Accepted 15 August 2011

Keywords:
Coupled systems
Model reduction
Structure preservation
Laguerre polynomials
Function approximation

ABSTRACT

In this paper, based on Laguerre polynomials, we present new methods for model reduction of coupled systems in the time domain. By appropriately selected projection matrices, a reduced order system is produced to retain the topology structure of the original system. Meanwhile, it preserves a desired number of Laguerre coefficients of the system's output, thereby providing good approximation accuracy. We also study the two-sided projection method in the time domain, as well as the stability of reduced order systems. Two numerical examples are used to illustrate the efficiency of the proposed methods.

1. Introduction

Coupled systems frequently arise in scientific and industrial fields [1]. A coupled system is a global model composed of several subsystems, which interact via additional relations between their local inputs and outputs. Coupled systems can be represented in terms of the global input–output behavior, while the architecture of the interconnection allows the inner subsystems to be still recognizable. The applications of coupled systems include models of the power grid, micro-electro-mechanical systems (MEMS) and domain decomposition methods of partial differential equations (PDE); see [2–5].

In this paper, we consider model reduction for coupled systems with special coupled relationships. Such systems consist of k coupled linear time-invariable (LTI) subsystems

$$\begin{cases} E_i \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} = A_i x_i(t) + B_i u_i(t), \\ y_i(t) = C_i x_i(t), \end{cases} \tag{1}$$

where $E_i, A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times p_i}$, $C_i \in \mathbb{R}^{m_i \times n_i}$; $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{p_i}$ and $y_i(t) \in \mathbb{R}^{m_i}$ are the state, input and output of the subsystems, respectively, $i = 1, 2, \ldots, k$. All subsystems are coupled through the linear algebraic relationships

$$\begin{aligned} u_i(t) &= K_{i1} y_1(t) + \cdots + K_{ik} y_k(t) + G_i u(t), \\ y(t) &= R_1 y_1(t) + \cdots + R_k y_k(t), \end{aligned} \tag{2}$$

where $K_{il} \in \mathbb{R}^{p_i \times m_l}$ $(l = 1, 2, \ldots, k)$, $R_i \in \mathbb{R}^{m \times m_i}$ and $G_i \in \mathbb{R}^{p_i \times p}$ are constant matrices. Further, $u(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^m$ are the input and the output of coupled systems, respectively. For simplicity, we assume zero initial conditions.

☆ This work was supported by the Natural Science Foundation of China (NSFC) under grant 11071192 and the International Science and Technology Cooperation Program of China under grant 2010DFA14700.

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0898-1221/$ – see front matter
doi:10.1016/j.camwa.2011.08.039

Generally, each subsystem possesses a lot of internal variables, which leads to the high dimensional coupled systems. As a result, direct numerical simulation in such large scale setting becomes an intractable task. Model reduction is an approach to overcoming this problem [6,7]. It aims to approximate a large scale system by a reduced one of lower order. Meanwhile, the topology structure of coupled systems should be preserved. Unfortunately, the topology structure is no longer recognizable after one applies standard model reduction methods to coupled systems directly.

Model reduction for coupled systems was originally considered in [8]. Recently, several researchers paid attention to this area and achieved some constructive results. In view of the topology structure of coupled systems, one can approximate each subsystem separately and then couple them with the same interconnection structure. However, a reduced order subsystem may require a high dimension to capture its individual behavior, but hardly affect the behavior of the whole coupled system. This means that such a strategy is not satisfactory. For this reason, the balanced truncation method resulting from the global response perspective was introduced in [9]. Furthermore, a priori error bound was obtained in [10] for a special case when coupled systems had the so-called "structured Gramians". This approach has been extended to more general cases in [11]. On the other hand, projection methods based on Krylov subspaces have been developed for coupled systems to match the moments in the frequency domain. We recommend the survey paper [12] for more details.

Model reduction for coupled systems in the time domain is studied in this paper. By projecting the time responses of each subsystem onto a lower dimensional subspace spanned by Laguerre coefficients, the reduced order coupled system matches these Laguerre coefficients and naturally has the same topology structure as the original system. Moreover, two-sided projection methods are used in this context to match more Laguerre coefficients, or moments in the frequency domain simultaneously. Our results can be viewed as a generalization of paper [13,14] to coupled systems. For simplicity, we only consider a special case of the non-scaled Laguerre functions. The standard model reduction techniques for the general LTI systems in the time domain can be found in [13–18].

The paper is organized as follows. In Section 2, based on Laguerre polynomials, we construct a reduced order system and its main properties on Laguerre coefficients preservation are proved rigorously. We employ two-sided projection methods to improve the proposed method. The stability of reduced order systems is also considered. In Section 3, two numerical examples are used to verify our theoretical results. We give some conclusions in Section 4.

2. Laguerre-based model reduction

In this section, we present a projection framework for constructing reduced order coupled systems with desired properties on the Laguerre coefficient preservation. Two-sided methods are employed to improve the approach. The connections with the results on moment matching methods will also be discussed. We first give a sketch of Laguerre polynomials.

2.1. Overview of Laguerre polynomials

Laguerre polynomials [19] $L_i : [0, \infty) \to \mathbb{R}$ are defined as

$$L_i(t) = \mathrm{e}^t \frac{\mathrm{d}^i}{\mathrm{d}t^i}(t^i \mathrm{e}^{-t}), \quad i = 0, 1, \ldots,$$

that is, $L_0(t) = 1, L_1(t) = 1 - t, L_2(t) = 2 - 4t + t^2, L_3(t) = 6 - 18t + 9t^2 - t^3, \ldots$. When weighted with the nonnegative function $w(t) = \mathrm{e}^{-t}$, Laguerre polynomials are orthogonal and satisfy the equation

$$\langle L_i(t), L_k(t)\rangle = \int_0^\infty w(t)L_i(t)L_k(t)\mathrm{d}t = \begin{cases} 0, & i \neq k, \\ (i!)^2, & i = k. \end{cases}$$

Additionally, Laguerre polynomials have the following recurrent relationships

$$L_0(t) = 1, \qquad L_1(t) = 1 - t, \quad \text{and} \quad L_{i+1}(t) = (1 + 2i - t)L_i(t) - i^2 L_{i-1}(t), \quad i = 1, 2, \ldots.$$

For our purposes, we prefer the following integral relationship

$$\int_0^t L_i(\tau)\mathrm{d}\tau = -\frac{1}{i+1}L_{i+1}(t) + L_i(t), \quad i = 0, 1, \ldots. \tag{3}$$

Provided that the k-th Laguerre approximation of the function $f(t)$ is

$$\hat{f}_k(t) = \hat{c}_0 L_0(t) + \hat{c}_1 L_1(t) + \cdots + \hat{c}_k L_k(t),$$

where Laguerre coefficients $\hat{c}_j = \frac{1}{(j!)^2}\int_0^\infty \mathrm{e}^{-t} f(t) L_j(t)\mathrm{d}t$ for $j = 0, 1, \ldots, k$, then $\hat{f}_k(t)$ is optimal in the sense that

$$\int_0^\infty \mathrm{e}^{-t}(f(t) - f_k(t))^2 \mathrm{d}t$$

is minimal subject to $f_k(t) = c_0 L_0(t) + c_1 L_1(t) + \cdots + c_k L_k(t)$ with $c_j \in \mathbb{R}$.

In the framework of model reduction, if we construct a reduced order system which preserves certain Laguerre coefficients of the system's output, then we expect it to approximate the original system in some way.