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Image segmentation based on histogram analysis utilizing the cloud model

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ABSTRACT

Both the cloud model and type-2 fuzzy sets deal with the uncertainty of membership which traditional type-1 fuzzy sets do not consider. Type-2 fuzzy sets consider the fuzziness of the membership degrees. The cloud model considers fuzziness, randomness, and the association between them. Based on the cloud model, the paper proposes an image segmentation approach which considers the fuzziness and randomness in histogram analysis. For the proposed method, first, the image histogram is generated. Second, the histogram is transformed into discrete concepts expressed by cloud models. Finally, the image is segmented into corresponding regions based on these cloud models. Segmentation experiments by images with bimodal and multimodal histograms are used to compare the proposed method with some related segmentation methods, including Otsu threshold, type-2 fuzzy threshold, fuzzy C-means clustering, and Gaussian mixture models. The comparison experiments validate the proposed method.

1. Introduction

In order to deal with the uncertainty of image segmentation, fuzzy sets were introduced into the field of image segmentation, and some methods were proposed in the literature. From the perspective of scene understanding and image interpretation, a segmentation method was proposed [1]. The method fits fuzzy membership functions to the modes of interest in histograms. A voxel which has a peak in a histogram is assigned the maximum confidence 1, and monotonically decreasing levels are assigned to the voxels on both sides of the peak. Image segmentation is performed based on fuzzy labels [1]. A histogram-based method to generate membership functions for extracting features of brain tissues on MRI (Magnetic Resonance Imaging) images is proposed [2], which detects the peak or valley features of the histogram, and then transforms the histogram to fuzzy membership functions corresponding to brain tissue types [2].

The methods based on fuzzy sets transform the histogram into corresponding membership degrees. They do not consider the uncertainty of membership functions and membership degrees. Zadeh notes this problem and proposes type-2 fuzzy sets [3]. Mendel makes a good contribution to the development of type-2 fuzzy sets [4]. An image thresholding method based on type-2 fuzzy sets has been proposed in [5]. Although these methods consider the fuzziness of membership functions and membership degrees, they do not consider the randomness. Randomness and fuzziness are the two most important uncertainties, and many concepts simultaneously contain randomness and fuzziness. Considering the fuzziness, randomness, and their association relationship, Li proposes the cloud model based on the normal distribution and the Gaussian membership function [6,7]. This uses a whole model including many discrete cloud drops to express a concept.

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Every observation point corresponds to multiple membership degrees, which fit a kind of probability distribution rather than a fixed number or a range. Based on the idea of considering fuzziness, randomness and their relationship, many techniques based on the cloud model are proposed, such as normal cloud generator, normal cloud transformation, normal cloud synthesis and uncertainty reasoning [6,8]. Recently, the cloud model has been successfully applied into some fields, such as traffic control, image segmentation, remote sensing image classification *etc.* [9–11].

The traditional image segmentation methods with uncertainty only consider randomness or only consider fuzziness. The image segmentation method based on the cloud model can consider randomness, fuzziness and their association. Based on the cloud model, the paper proposes an image segmentation method which transforms histograms into cloud models with uncertain membership degrees; the image segmentation is realized by the extraction of concepts, which are expressed by normal cloud models. The method first transforms the feature points (valleys and peaks) of histogram into membership degrees, then uses the objective function to transform the histogram between two near valley points into corresponding cloud drops. The method uses the whole cloud model to express a concept. Each concept corresponds to an image class. Each pixel is assigned to an image class with maximum membership degree based on the maximum discriminant principle. Image segmentation is realized last. In the end, the paper compares the proposed method with some related segmentation methods based on Otsu threshold, type-2 fuzzy threshold, fuzzy C-means clustering (FCM), and Gaussian mixture models (GMMs). The experimental results validate the proposed method.

The remainder of the paper is organized as follows. Section 2 introduces the basic principles of the cloud model. Section 3 describes the proposed method of image segmentation. The experimental analysis and conclusions are presented in Sections 4 and 5, respectively.

2. Basic principles of the cloud model

There are various implementation approaches of the cloud model, resulting in different kinds of cloud models. The normal cloud model is the most commonly used model, which is based on the normal distribution and the Gaussian membership function. It can be described as follows.

Let U be a quantitative universal set and C be the qualitative concept related to U. If $x \in U$, which is a random realization of the concept C, and x satisfies $x \sim N(Ex, En'^2)$, where $En' \sim N(En, He^2)$, and the certainty degree of x on C is

$$\mu = \exp\left[-\frac{(x-Ex)^2}{2(En')^2}\right] \tag{1}$$

then the distribution of x on U is a normal cloud [7], and every x is defined as a cloud drop.

The normal cloud model employs the expected value Ex, the entropy En, and the hyper-entropy He to represent the concept. Ex is the mathematical expectation of the cloud drops distributed in the universal set. En is the uncertainty measurement of the qualitative concept. From the perspective of probability theory, it is similar to standard variance of random variables. From the point of view of fuzzy set theory, it represents the value scope which the drop is acceptable by the concept, and it defines the support set of the concept with membership degrees larger than 0. As a result, the correlation of randomness and fuzziness is reflected by the same numerical character. He is the uncertainty measurement of the entropy En, i.e., the second-order entropy of the entropy [6].

Given the three parameters Ex, En, He, the normal cloud model can be generated [6].

Input: Ex, En, He, and the number of the cloud drops n.

Output: n of cloud drops x and their degree μ, i.e., drop (x_i, μ_i), $i = 1, 2, \dots, n$.

Step 1. Generate a normally distributed random number En'_i with expectation En and variance He^2, i.e.,$En'_i = \mathrm{NORM}(En,\ He^2)$.

Step 2. Generate a normally distributed random number x_i with expectation Ex and variance $En_i'^2$, i.e., $x_i = \mathrm{NORM}(Ex, En_i'^2)$.

Step 3. Calculate

$$\mu_i = \exp\left[-\frac{(x_i-Ex)^2}{2(En'_i)^2}\right]. \tag{2}$$

Step 4. x_i with certainty degree of μ_i is a cloud drop in the domain.

Step 5. Repeat Steps 1 to 4 until n cloud drops are generated.

For example, the concept "number near 25" expressed by cloud C (25, 3, 0.3), $n = 10{,}000$ is illustrated in Fig. 1.

Traditional type-1 fuzzy sets do not consider the uncertainty of membership functions. To deal with the problem, Zadeh proposes type-2 fuzzy sets, the extension of ordinary fuzzy sets [3]. The membership degrees of such sets themselves are type-1 fuzzy sets [12]. Gaussian membership function is the most widely-used one in fuzzy sets, and generally represented as

$$\mu_A(x) = \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]. \tag{3}$$