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A two-dimensional lattice Boltzmann model for uniform channel flows

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ABSTRACT

In this paper a novel two-dimensional lattice Boltzmann model (LBM) is developed for uniform channel flows. The axial velocity is solved from a momentum diffusion equation over the cross-sectional plane. An extrapolation boundary condition is also introduced to enhance the no-slip boundary in the momentum equation. This boundary treatment can also be applied to LBM simulations of other diffusion processes. The algorithm and boundary treatment are validated by simulations of steady Poiseuille and pulsatile Womersley flows in circular pipes. The numerical convergence and accuracy are comparable to those of existing models. Moreover, comparison with general three-dimensional lattice Boltzmann simulations demonstrates the advantages of our two-dimensional model, including lower computational resource requirements (memory and time), easier boundary treatment for arbitrary cross-sectional shapes, and no velocity constraint. These features are attractive for practical applications with uniform channel flows.

1. Introduction

The lattice Boltzmann method (LBM) is a relatively new simulation technique for complex fluid systems [1–3]. Originating from classical statistical physics, LBM is a mesoscopic method, in which the fluid is modeled as a collection of pseudo-particles, and such particles propagate and collide over a discrete lattice domain. Macroscopic continuity and momentum equations can be obtained from this propagation–collision dynamics through a rigorous mathematical analysis. The particulate nature and local dynamics provide advantages for complex boundaries and parallel computation. Successful LBM applications include, to name but a few, those for multiphase flows, biological flows, particulate flows, flows in porous materials, solid–fluid interfacial phenomena, and electrokinetics and electrohydrodynamics.

Uniform channel flows are commonly found in many industrial applications. In such situations, the transverse velocity is zero and the axial velocity is independent of the axial position. Traditional numerical methods can easily simplify such problems by solving them over a two-dimensional (2D) domain, i.e., the channel cross-section. This greatly reduces the computational demand. However, due to the particular discretization of space and velocity in LBM, three-dimensional (3D) lattice structures are required to resolve the axial velocity and the cross-sectional shapes even for such uniform channel flows [4,5]. In addition to the large computation demand, such an approach also raises difficulties in the boundary treatment for curved surfaces. This has been a weak point of this attractive LBM algorithm when compared to other traditional computational fluid dynamics methods. Several axisymmetric LBM models [6–9] have been proposed; however, their applications are limited to circular pipes and are no help for channels with arbitrary, non-circular cross-sectional shapes.

Therefore, in this work, we propose to solve the governing Navier–Stokes equation of fluid dynamics in uniform channel flows as a convection–diffusion equation for the axial momentum over a 2D lattice space. This algorithm has been proved by our numerical results with a much lower computational demand (time and memory), easier boundary treatment, and comparable accuracy as compared to the commonly used 3D LBM models. In addition, the low velocity limit in the general

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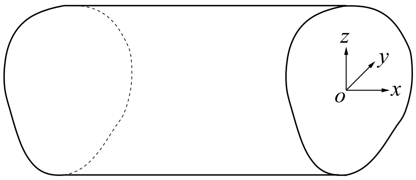

Fig. 1. Schematic of a uniform channel with arbitrary cross-sectional shape.

LBM algorithm has been removed. This could be advantageous for reducing the computational time with a larger time step and for improving the simulation accuracy with a larger velocity range.

2. Theory and model

2.1. Uniform channel flows

In general, the dynamic behaviors of incompressible flows are governed by the continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial v}+\frac{\partial w}{\partial z}=0 \tag{1}$$

and the Navier–Stokes equations [10]

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right). \tag{4}$$

Here, t is time, $\mathbf{g}=(g_x, g_y, g_z)$ is the body force, P is the pressure, ρ is the density, and μ is the viscosity. u, v, and w are the three velocity components in the x, y, and z directions, respectively, in the Cartesian coordinates. For the particular situation of flows through long, straight, and uniform channels (Fig. 1), there is no transverse velocity in the cross-sectional plane, i.e.,

$$v=w=0 \tag{5}$$

and, according to hydrostatics, the pressure in the cross-sectional plane is adjusted to counterbalance the external force [10]

$$-\frac{\partial P}{\partial y}+\rho g_y=-\frac{\partial P}{\partial z}+\rho g_z=0. \tag{6}$$

In addition, the axial velocity u now is independent of the axial location x:

$$u=u(y,z,t). \tag{7}$$

Under such conditions, Eqs. (1), (3) and (4) are automatically satisfied and Eq. (2) is simplified to

$$\rho\frac{\partial u}{\partial t}=F_x+\mu\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{8}$$

where

$$F_x=-\frac{\partial P}{\partial x}+\rho g_x. \tag{9}$$

Substituting the fluid momentum

$$n=\rho u \tag{10}$$

into Eq. (8) and taking $\rho=$ constant for incompressible flows yields

$$\frac{\partial n}{\partial t}=F_x+\nu\left(\frac{\partial^2 n}{\partial y^2}+\frac{\partial^2 n}{\partial z^2}\right), \tag{11}$$

where $\nu=\mu/\rho$ is the kinematic viscosity. The above equation can also be considered as a diffusion equation for the axial momentum n, with F_x as a source term and ν as the diffusion coefficient. Actually ν is also often interpreted as the momentum diffusion coefficient in fluid mechanics [10].

2.2. The lattice Boltzmann method for solving the diffusion equation

In addition to its applications in simulating fluid systems, LBM has also been employed as a differential equation solver for other problems, such as those of heat transfer [11], electrical fields [12–14], and convection–diffusion processes [15–18].