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Lattice Boltzmann large eddy simulation of subcritical flows around a sphere on non-uniform grids

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ABSTRACT

In this work, the suitability of the lattice Boltzmann method is evaluated for the simulation of subcritical turbulent flows around a sphere. Special measures are taken to reduce the computational cost without sacrificing the accuracy of the method. A large eddy simulation turbulence model is employed to allow efficient simulation of resolved flow structures on non-uniform computational meshes. In the vicinity of solid walls, where the flow is governed by the presence of a thin boundary layer, local grid-refinement is employed in order to capture the fine structures of the flow. In the test case considered, reference values for the drag force in the Reynolds number range from 2000 to 10 000 and for the surface pressure distribution and the angle of separation at a Reynolds number of 10 000 could be quantitatively reproduced. A parallel efficiency of 80% was obtained on an Opteron cluster.

1. Introduction

In recent years, the lattice Boltzmann method (LBM) has been established as a method for solving weakly compressible and incompressible flow problems. The LBM was found to be competitive with traditional methods based on the direct discretization of the Navier–Stokes equations in a wide range of applications with complex flow physics, including multiphase flows, suspensions in fluids, and flows in porous media [1–3]. LBM has also been demonstrated to be an efficient simulation tool for laminar flows [4]. However, the efficiency of LB methods for flow problems around bluff bodies in the turbulent subcritical regime has not been investigated in depth, which may be due to the fact that parallel LBM implementations based on turbulence models on non-uniform hierarchical grids require substantial programming effort. Yet, the efficient simulation of turbulent engineering problems requires the use of non-uniform meshes to resolve the large velocity gradients in the boundary layer. As a direct numerical simulation (DNS) of technically relevant turbulent flows is prohibitively expensive, turbulence modeling is required. Two main types of turbulence models have previously been introduced into the LBM context: Reynolds averaged Navier–Stokes (RANS) type models [5] and large eddy simulation models (LES) [6]. Additionally, hybrid approaches have been developed, such as the detached Eddy simulation [7]. In RANS models, a time- or ensemble-averaged velocity is computed and the turbulent features of the flow have to be modeled, including all of the spatial scales. LES models, on the other hand, apply a spatial low-pass filter to the hydrodynamic fields. The optimal filter would eliminate only high frequencies in the flow field while their influence on the large eddies, which are resolved in the simulation, is modeled. However, the optimal filter could only be defined in Fourier space and, hence, would require prohibitively large stencils. This is true for the lattice Boltzmann model and a classical finite difference Navier–Stokes solver alike. However, the simplest possible filter based on shear stress can be implemented very efficiently in the lattice Boltzmann method because of the local availability of the shear stress in the non-equilibrium part of the distribution

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function. This information can be employed to implement an implicit local low-pass filter also for non-uniform grids at very low computational cost.

In this article, an efficient implementation of a lattice Boltzmann method with LES turbulence modeling on non-uniform grids is introduced, which allows a relatively simple turbulence modeling to be successfully applied to problems relevant in engineering. The first part of this article will briefly recall some basic features of the lattice Boltzmann multiple-relaxation-time method with a subgrid stress model. The second part will deal with extensions of the method for non-uniform grids. In the third part, the method will be applied to the calculation of some distinct flow features of a sphere in a channel in the subcritical regime.

2. Lattice Boltzmann method with subgrid stress model (LBM-SGS)

Unlike traditional numerical methods in Computational Fluid Dynamics that apply some discretization technique to the Navier–Stokes equations, the lattice Boltzmann method is a discretization of the Boltzmann equation in a discrete velocity space. Its solutions can be shown to converge asymptotically to the solution of the incompressible Navier–Stokes equation [8]. The starting point for the derivation of the lattice Boltzmann method is a momentum density distribution of identical particles, $f(\mathbf{x}, \boldsymbol{\xi}, t)$. The distribution is discrete in space, velocity space, and time. In the lattice Boltzmann method the discretization of space, called the lattice, corresponds exactly to the discretization of velocity space, so that a virtual particle sitting on a node in the lattice moves to a neighboring node in a discrete time step. Hence, interpolation is not required. The Navier–Stokes equation is found to be the equation of motion for the first moment $\sum_i \mathbf{e}_i f_i$ of the momentum distribution, provided that a sufficiently isotropic velocity set $\mathbf{e}_i$ is used. A common choice for the lattice Boltzmann velocity set is the D3Q19 (three dimensions, 19 speeds, cf. Appendix B) model [8]. Its microscopic velocity components correspond to the third, fifth and seventh rows of the matrix given in Appendix C. The evolution of the lattice Boltzmann model can be split into two steps: first, free streaming or propagation of the distribution f_i according to its respective speed and, second, collision on the lattice nodes. In order to obtain the Navier–Stokes equation, the collision operator has to fulfill some conditions concerning conservation laws, Galilean invariance, and dissipation. Still, the collision operator is not uniquely defined. Different choices will have different stability characteristics. In this paper, a version of a multiple-relaxation-time [9] collision operator is employed. That specific version was used in previous works of the authors (e.g. [10]) and the description is recapped here.

The lattice Boltzmann model can be written as

$$f_i(t + \Delta t, \mathsf{x} + \mathsf{e}_i \Delta t) = f_i(t, \mathsf{x}) + \Omega_i, \quad i = 0, \ldots, b-1, \tag{1}$$

where Δt is the time step and the collision operator is given by

$$\boldsymbol{\Omega} = \mathsf{M}^{-1}\mathsf{S}\left[(\mathsf{M}\boldsymbol{f}) - \boldsymbol{m}^{\mathrm{eq}}\right]. \tag{2}$$

Matrix M is the transformation matrix given in Appendix C, composed of the 19 orthogonal basis vectors $\{\boldsymbol{\Phi}_i,\ i = 0, \ldots, b-1\}$ given in Appendix A, which are orthogonal with respect to a weighted inner product, $\langle \boldsymbol{\Phi}_i, \boldsymbol{\Phi}_j \rangle_w = \sum_k \Phi_{ik}\Phi_{jk} w_k = 0$, if $i \neq j$ (in contrast to [9], where $\langle \boldsymbol{\Phi}_i, \boldsymbol{\Phi}_j \rangle = \sum_k \Phi_{ik}\Phi_{jk} \neq 0$, if $i \neq j$). The vector $\boldsymbol{w}$ is composed of the weights $\{w_i,\ i = 0 \ldots, b-1\}$:

$$\boldsymbol{w} = \left(\frac{1}{3}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{18}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}, \frac{1}{36}\right).$$

The moments $\boldsymbol{m} = \mathsf{M}\boldsymbol{f}$ are labeled as

$$\boldsymbol{m} = (\rho, e, \epsilon, j_x, q_x, j_y, q_y, j_z, q_z, 3p_{xx}, 3\pi_{xx}, p_{ww}, \pi_{ww}, p_{xy}, p_{yz}, p_{xz}, m_x, m_y, m_z).$$

$\boldsymbol{m}^{\mathrm{eq}}$ is the vector composed of the equilibrium moments given in Eq. (5) and $\mathsf{S} = \{s_{ii},\ i = 0, \ldots, b-1\}$ is the diagonal collision matrix. The non-zero collision parameters s_{ii} (the eigenvalues of the collision matrix $\mathsf{M}^{-1}\mathsf{S}\mathsf{M}$) are:

$$\begin{aligned}
&s_{1,1} = s_a \\
&s_{2,2} = s_b \\
&s_{4,4} = s_{6,6} = s_{8,8} = s_c \\
&s_{10,10} = s_{12,12} = s_d \\
&s_{9,9} = s_{11,11} = s_{13,13} = s_{14,14} = s_{15,15} = -\frac{\Delta t}{\tau} = s_\omega \\
&s_{16,16} = s_{17,17} = s_{18,18} = s_e.
\end{aligned} \tag{3}$$

The relaxation time τ is chosen as

$$\tau = 3\frac{\nu}{c^2} + \frac{1}{2}\Delta t, \tag{4}$$

where ν is the kinematic viscosity. The parameters s_a, s_b, s_c, s_d and s_e can be freely chosen in the range $[-2, 0]$ and tuned to improve stability [11]. While the optimal values for these parameters depend on both the geometry and the