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# A viscosity counteracting approach in the lattice Boltzmann BGK model for low viscosity flow: Preliminary verification

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ABSTRACT

Due to numerical instability, the lattice Boltzmann model (LBM) with the Bhatnagar–Gross–Krook (BGK) collision operator has some limitations in the simulation of low viscosity flows. In this paper, we propose a viscosity counteracting approach for simulating a moderate viscosity flow. An extra negative viscosity term is introduced to counteract part of the moderate viscosity by using the lattice Boltzmann equation with a source term. The counteracting viscosity term is treated as a non-uniform unsteady source. The stability is enhanced; thus small viscosity flows can be simulated. Model verification consists of benchmark cases such as those of Poiseuille flow, Couette flow, waterhammer waves, Taylor–Green vortex flow, and lid-driven cavity flow. The flow patterns, error characteristics, and representative parameters are carefully analyzed. It is shown that this approach can simulate flows with lower viscosities than may be simulated using the normal LBGK model; the second-order accuracy of the LBGK model is definitely retained, although a little dissipation is added. These preliminary studies prove the effectiveness and accuracy of the model. Sophisticated analysis and further verification of the stability mechanism will be done in the near future.



## 1. Introduction

The lattice Boltzmann method (LBM) is a powerful technique for computational modeling of a wide variety of complex flow problems [1]. Among the many LBM models, the model featuring the Bhatnagar–Gross–Krook (BGK) collision operator (generally labeled as LBGK, for the lattice Boltzmann BGK model, or SRT–LBM, for the single-relaxation-time LBM model) [2] is very popular because of its simple formulation and convenient application. Nevertheless, the LBGK model has some difficulties in simulating high Reynolds number flow, owing to numerical stability problems in a low viscosity regime. Though the mechanism of LBM's instability is not totally understood, it is normally attributed to the occurrence of unphysical negative distribution functions, the interplay between acoustic modes and other modes in a low viscosity regime, improper treatments of boundary conditions, etc. [3–6]

Many efforts to improve LBM's stability or to simulate low viscosity flows have been made. McNamara et al. [7] applied the Lax–Wendroff scheme to enhance the stability of thermal LBM. Qian [8] used upwind interpolation in the fractional propagation LBM to suppress staggered invariants. Dellar [9] proposed adjusting the bulk viscosity, independently from the shear viscosity, for better numerical stability. Lallemand et al. [10] and d'Humières et al. [11] developed the multiple-relaxation-time models (MRT–LBM) to allow the separation of the relaxations of the various physical and kinetic modes,

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obtaining quite good stability features. Ansumali et al. [12] proposed the entropy function based LBM (ELBM) and declared it to have better stability. Li et al. [5] imposed a lower bound on the relaxation times to ensure positivity of the distributions. Shu et al. [13] proposed a fractional step LBM scheme for incompressible high Reynolds number flows. Fan et al. [14] introduced some hyperviscosities to add numerical dissipation into the model. Tosi et al. [15,16] proposed an H-theorem compliant ELBM by adjusting the local relaxation time of the standard LBM and compared the entropic schemes versus positivity-enforcing schemes. Brownlee et al. [17–20] proposed positivity preservation, non-equilibrium entropy limiters, and the Ehrenfest coarse-graining regularization to improve the stability. Niu et al. [21] compared the stability features of the differential LBM (DLBM) [22], the interpolation-supplemented LBM (ISLBM) [23], and the Taylor-series-expansion and least-square based LBM (TLLBM)[24], and concluded that the ISLBM and TLLBM improve the numerical stability by increasing hyperviscosities. Most recently, Ricot et al. [6] proposed spatial filtering on the LBM equation or macroscopic quantities to eliminate spurious fluctuations. Chen et al. [25] compared the existing four LBM models (LBGK, ELBM, DLBM and MRT–LBM) and proved that MRT–LBM is the best in accuracy, stability, and efficiency. These studies are very insightful for understanding the instability mechanism and could provide guidance for simulations of low viscosity flows. However, a simple but effective approach for enhancing LBM's stability is still needed.

In this paper, we propose a viscosity counteracting approach for improving the LBGK model's stability in a low viscosity regime. We will give a brief description of the approach in Section 2, verify it using benchmark cases in Section 3, and finally conclude the paper in Section 4.

## 2. Methods

To simulate high Reynolds number flows in finite lattice resolutions, we need to reduce the viscosity to as low a value as possible. But in small viscosity conditions, instability frequently occurs. Therefore, any means that can reduce viscosity without introducing instability and additional error is valuable.

The idea of the viscosity counteracting approach is based on the following consideration. The viscosity term in the Navier–Stokes (N–S) equations is the second-order derivative term. It acts as a dissipation factor and maintains numerical stability. When the N–S equations are solved by a numerical scheme, the higher order truncation errors also play important roles in the stability and accuracy. The widely used D2Q9 and D3Q19 models are second-order schemes in space, and some changes of the third-order or higher terms of truncation errors may be beneficial to their stability.

Therefore, the N–S equations may be expressed as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0, \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + (\nu + \nu_c)\frac{\partial}{\partial x_j}\left(2\rho S_{ij}\right) - \nu_c \frac{\partial}{\partial x_j}\left(2\rho S_{ij}\right), \tag{2}$$

in which $\nu + \nu_c$ is the viscosity modeled by the LBGK equation, $-\nu_c$ is the counteracting viscosity, and $S_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$. The counteracting term $-\nu_c \frac{\partial}{\partial x_j}\left(2\rho S_{ij}\right)$ will be treated as a forcing term. When a stable viscosity $\nu + \nu_c$ is properly chosen, the simulation results will correspond to the viscosity $\nu$.

To model the inclusion of the counteracting term in the above N–S equations, the LBGK equation should be able to introduce the forcing term. Here we use the LBGK equation with a source term, as follows [26]:

$$f_\alpha(\boldsymbol{x} + \boldsymbol{e}_\alpha \delta_t, t + \delta_t) - f_\alpha(\boldsymbol{x}, t) = -\frac{1}{\tau}[f_\alpha(\boldsymbol{x}, t) - f_\alpha^{eq}(\boldsymbol{x}, t)] + \frac{\delta_t}{2}[g_\alpha(\boldsymbol{x}, t) + g_\alpha(\boldsymbol{x} + \boldsymbol{e}_\alpha \delta_t, t + \delta_t)], \tag{3}$$

where $f_\alpha$ is the particle velocity distribution function along the $\alpha$th particle velocity direction $\boldsymbol{e}_\alpha$, $f_\alpha^{eq}$ the equilibrium distribution function, $g_\alpha$ the forcing term function, $\tau$ the relaxation factor, $\boldsymbol{e}_\alpha$ the discrete particle vector, $\boldsymbol{x}$ the lattice grid, and $\delta_t$ the time increment.

The forcing term function $g_\alpha$ may take the following form to guarantee second-order convergence for a non-uniform and unsteady body force and a mass source in the N–S equations [26]:

$$g_\alpha = w_\alpha \left\{A + 3\boldsymbol{B} \cdot [(\boldsymbol{e}_\alpha - \boldsymbol{u}) + 3(\boldsymbol{e}_\alpha \cdot \boldsymbol{u})\boldsymbol{e}_\alpha]\right\}, \tag{4}$$

in which $A$ is the source term in the fluid continuity equation, $B$ the external forcing term in the momentum equation, and $w_\alpha$ the weighting parameter of distribution functions.

For Eqs. (1) and (2), we just let

$$A = 0, \qquad B_i = -\nu_c \frac{\partial}{\partial x_j}\left(2\rho S_{ij}\right). \tag{5}$$

The selection of this method of introducing the forcing term is essential for the feasibility of the proposed approach. Because the counteracting term is a second-order term, the method of introducing the forcing term must be at least a second-order one. Otherwise, additional second-order dissipations would be added and the results would not be as anticipated.