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Reduced order modeling based on POD of a parabolized Navier–Stokes equations model II: Trust region POD 4D VAR data assimilation

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ABSTRACT

A reduced order model based on Proper Orthogonal Decomposition (POD) 4D VAR (Four-dimensional Variational) data assimilation for the parabolized Navier–Stokes (PNS) equations is derived. Various approaches of POD implementation of the reduced order inverse problem are studied and compared including an ad-hoc POD adaptivity along with a trust region POD adaptivity. The numerical results obtained show that the trust region POD 4D VAR provides the best results amongst all the POD adaptive methods tested in all error metrics for the reduced order inverse problem of the PNS equations.

1. Introduction

The parabolized Navier–Stokes (PNS) equations are simplified Navier–Stokes equations obtained by eliminating the streamwise second order viscous terms [1,2]. The solution can be obtained by marching in the streamwise direction (i.e. in the x direction along the surface, downstream direction) from some known initial location. Thus the x direction is taken as time and the y direction is taken as space for a two-dimensional problem, which makes the problem a one-dimensional problem in space actually.

The four-dimensional variational (4D VAR) data assimilation process seeks the minimum of a functional estimating the discrepancy between the solution of the model and the observation [3]. The derivation of the optimality system, using the adjoint model, permits us to compute a gradient which is used in the optimization.

The data assimilation problem, which is one type of inverse computational fluid dynamics (CFD) problems, is characterized by the high CPU time and memory load required for the computation of the cost functional and its gradient, as well as by the instability (due to ill-posedness) which prohibits use of Newton-type algorithms without prior explicit regularization [1]. Specifically, the computation of the gradient of the cost functional with respect to the control variables using the adjoint model requires the same computational effort as the direct model.

For the data assimilation problem of the PNS equations, the POD model reduction technique [4,5] for the introduction of the POD theory and [6–10] for the application of POD is introduced in order to improve the efficiency of the 4D VAR data assimilation process [11].

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Since the validity of the POD reduced order model is limited to the vicinity of the design parameters in the control parameter space, it might not be an appropriate model when the latest state is significantly different from the one on which the POD reduced order model is based. Therefore, an 'ad-hoc' adaptive POD 4D VAR data assimilation method [12,13,3] was implemented by updating the POD reduced order model during the optimization process.

To improve the performance of the 'ad-hoc' adaptive POD 4D VAR data assimilation method, the trust region POD 4D VAR data assimilation was introduced by Bergmann and Cordier [14] and Arian et al. [15]. It was applied to fluid mechanics for the first time by Fahl [16] in a flow control problem with the unsteady boundary condition being the control variables. In the data assimilation process of the PNS model, the initial condition is used as the control variable.

Combining the POD model reduction technique with the concept of the trust region optimization method [17] presents a framework for deciding when to update the POD reduced order model by projecting back to the high-fidelity model during the optimization process [18,16,15]. The limited-memory BFGS (L-BFGS) quasi Newton optimization method was used in the minimization of the cost function. Moreover, the trust region method is supported by a global convergence result that ensures the trust region iterates produced by the optimization algorithm that started at an arbitrary initial iterate, will converge to a local optimizer of the high-fidelity 4D VAR problem [19,18,16,15].

Part I of this paper relates to reduced order modeling based on POD of a PNS equations model and is focused on the POD reduced order forward model. The POD 4D VAR data assimilation process performed in this paper is based on the POD reduced order forward model. During the adaptive POD 4D VAR data assimilation process, a new set of snapshots is generated from the full forward PNS model using an updated initial condition (control variable). The reduced order forward model was then updated using the new set of snapshots.

In the present article we apply the POD method to derive a reduced order model of the data assimilation problem for the PNS equations and then introduce the POD 4D VAR adaptivity to improve the performance of the reduced order model. The trust region scheme is combined with POD 4D VAR data assimilation in order to solve the reduced order inverse problem more efficiently. To the best of our knowledge, this is a first application of the POD 4D VAR and the adaptive POD 4D VAR (the ad-hoc adaptive POD 4D VAR and the trust region POD 4D VAR) for a data assimilation problem addressing the PNS equations.

The paper is organized as follows. Section 2 presents the PNS model description along with the corresponding adjoint model of the PNS equations. Section 3 details the construction of the POD 4D VAR data assimilation model, consisting of Section 3.1 where the basic theory of the POD method is presented and Section 3.2 which illustrates the process of applying the POD method to the 4D VAR data assimilation of the inverse PNS model along with the algorithm of the ad-hoc adaptive POD 4D VAR method. Section 4 presents the classical trust region optimization method and the trust region scheme for the POD 4D VAR data assimilation. In Section 5 we present numerical results obtained comparing the performance of the POD 4D VAR, the ad-hoc adaptive POD 4D VAR and the trust region POD 4D VAR with that of the full 4D VAR for solving the inverse problem of the PNS equations. In Section 6 a summary and conclusions are provided including a discussion related to future research work.

2. PNS model description

2.1. Forward model

The two-dimensional steady supersonic laminar flow is modeled by the parabolized Navier–Stokes equations (PNS). This model is valid if the flow is supersonic along the x coordinate and the second order viscous effects along this direction are negligible, a fact which allows a rapid decrease in the computational time required to complete the calculation [20]. As a matter of fact, the x direction is taken as time and the y direction is taken as space when solving the equations numerically. The model description used here can be referred to Alekseev's works on the PNS equations [21–23]. The following equations describe an under-expanded jet (Fig. 1).

$$\begin{cases} \dfrac{\partial(\rho u)}{\partial x} + \dfrac{\partial(\rho v)}{\partial y} = 0 \\ u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = \dfrac{1}{Re\rho}\dfrac{\partial^2 u}{\partial y^2} \\ u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial y} = \dfrac{4}{3Re\rho}\dfrac{\partial^2 v}{\partial y^2} \\ u\dfrac{\partial e}{\partial x} + v\dfrac{\partial e}{\partial y} + (\kappa-1)e\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) = \dfrac{1}{\rho}\left(\dfrac{\kappa}{RePr}\dfrac{\partial^2 e}{\partial y^2} + \dfrac{4}{3Re}\left(\dfrac{\partial u}{\partial y}\right)^2\right) \\ p = \rho RT,\ e = C_v T = \dfrac{R}{(\kappa-1)T}, \quad (x,y)\in\Omega = (0<x<x_{\max}, 0<y<1) \end{cases} \tag{2.1}$$