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# The min–max split delivery multi-depot vehicle routing problem with minimum service time requirement



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The min-max Split Delivery Multi-Depot Vehicle Routing Problem with Minimum Service Time Requirement (min-max SDMDVRP-MSTR) is a variant of the Multi-Depot Vehicle Routing Problem. Each customer requires a specified amount of service time. The service time can be split among vehicles as long as each vehicle spends a minimum amount of service time at a customer. The objective is to minimize the duration of the longest route (where duration is the sum of travel and service times).

We develop a heuristic (denoted by MDS) that solves the min-max SDMDVRP-MSTR in three stages: (1) initialize a feasible solution without splits; (2) improve the longest routes by splitting service times; (3) ensure all minimum service time requirements are satisfied. The first stage of MDS is compared to an existing heuristic to solve the min-max Multi-Depot Vehicle Routing Problem on 43 benchmark instances. MDS produces 37 best-known solutions. We also demonstrate the effectiveness of MDS on 21 new instances whose (near) optimal solutions can be estimated based on geometry. Finally, we investigate the savings from split service and the split patterns as we vary the required service times, the average number of customers per route, and the minimum service time requirement.

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#### 1. Introduction

The classical Vehicle Routing Problem (VRP) models the distribution of goods from a single depot to the customers. A customer has a demand that must be satisfied in full by one visit of a vehicle. The sum of the demands delivered by a vehicle cannot exceed its capacity. A vehicle must start and end its route at the depot. There is usually no constraint on the number of vehicles used. The objective is to minimize the total distance traveled by all vehicles. The VRP was introduced by Dantzig and Ramser [7] in 1959 to model gasoline delivery. Many variants of the VRP have been developed to model real-world problems. We refer interested readers to Golden et al. [9] and Toth and Vigo [19,20] for comprehensive surveys of the VRP and its variants.

While most of the published research focuses on minimizing the sum of the route costs, minimizing the maximum route cost is applicable in situations where the last delivery is crucial or the balance of the route lengths is desired. Last delivery applications include military operations, disaster relief routing, newspaper

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delivery, and computer networks. Balancing route length applications include school bus routing and workload balance among drivers. Campbell et al. [4] and Bertazzi et al. [3] showed that, from the worst-case perspective, a solution to the min-max objective can be very different from the solution to the traditional min-sum objective. This finding motivates the development of exact and heuristic algorithms specifically designed for the min-max objective. Carlsson et al. [5] first proposed the min-max Multi-Depot VRP and solved it using a linear program-based, load balancing approach [23] and a region partitioning approach. Wang et al. [21] developed a three-stage heuristic (denoted by MD) that combined local search and perturbation strategies and improved the results of Carlsson et al. [5] significantly. Narasimha et al. [14] constructed an ant colony procedure to solve both the multi-depot and single-depot versions of the min-max problem. Ren [15] proposed a hybrid genetic algorithm for the single-depot minmax VRP.

Recently, Yakici and Karasakal [22] studied a min-max service VRP with split delivery and heterogeneous demands. Customer demands are described by the service times and the service types that are required. Customer service can be split among vehicles if it improves the min-max objective. If there is no route duration constraint, service times do not alter the routing plan of the classic VRP solution, but can change the routing plan of the min-max solution [3]. Split delivery often reduces the total cost to the carrier [1,2,8], but can inconvenience the customers because of work disruptions and paperwork. Gulczynski et al. [10] introduced a split delivery VRP with minimum delivery amounts. A customer's demand can be satisfied by multiple visits, provided each delivery is not less than a specified fraction of total demand.

In this paper, we study the min-max Split Delivery Multi-Depot Vehicle Routing Problem with Minimum Service Time Requirement (min-max SDMDVRP-MSTR). The objective function value has two components: (1) the travel times of the vehicles and (2) the service times of the customers on the route. They contribute differently to the objective. When the minimum delivery fraction is greater than one half, i.e., no split deliveries allowed, the service time contribution is determined entirely by individual customers. However, the travel time contribution is determined by all customers. In particular, if a new customer is added to the route, the increase in service time can be readily obtained, but the exact increase in travel time cannot be computed easily. When travel times dominate, the problem is closer to the min-max MDVRP studied by Carlsson et al. [5]. When service times dominate, the problem is closer to the Multi-Way Number Partitioning Problem [13]. Both problems are difficult to solve. When the travel times and service times are comparable, the problem represents a trade-off between the two equally weighted objectives.

We develop a heuristic algorithm (denoted by MDS) to solve the min-max SDMDVRP-MSTR. The MD solver developed by Wang et al. [21] is modified to generate a good initial solution without splits. Next, a network flow model is used to improve the solution by splitting service, assuming no minimum service time requirement. Finally, a linear program is solved to ensure that each visit by a vehicle has at least the minimum service time.

There are numerous potential applications of the SDMDVRP-MSTR including military operations, disaster relief, and the distribution of industrial gases and other products where the delivery service time is relatively large.

The remainder of the paper is organized as follows. In Section 2, the min–max SDMDVRP-MSTR is described formally. In Section 3, structural properties of the optimal solution when the minimum service time fraction is zero are provided. In Section 4, a heuristic algorithm (MDS) for the general problem is developed. In Section 5, the computational results are presented and discussed. Finally, Section 6 gives our concluding remarks.

#### 2. Problem description

Let  $\mathcal{G}(W \cup V, E)$  be a complete graph, where  $W = \{w_1, w_2, ..., v_n\}$  $w_{m-1}, w_m$  and  $V = \{v_1, v_2, \dots, v_{n-1}, v_n\}$  are two sets of vertices, and E is the corresponding set of edges. A vertex,  $w_i \in W$ , where j = 1, 2, ..., m, corresponds to a depot where a fixed number,  $l_i$ , of vehicles are stationed. A vehicle that starts from  $w_i$  must return to  $w_i$ at the end of its route. Unlike the classic min-sum VRP, which seldom specifies a finite number of vehicles, the min-max problem requires the number of vehicles in advance; otherwise the optimal solution will consist entirely of routes serving only one customer. A vertex,  $v_i \in V$ , where i = 1, 2, ..., n, corresponds to a customer who requires a service time of s<sub>i</sub>. A customer can be visited multiple times by different vehicles as long as the service requirement is met in full at the end of the last visit and each visit delivers the minimum required service time. An edge  $e \in E$  is associated with a cost,  $t_e$ , representing the travel time between the two vertices that define the edge. We assume that the travel times satisfy the triangle inequality. The total cost of a route, or its duration T, is the sum of the travel times spent on the road and the service times spent at the customers. Unlike the classic min-sum VRP, which often poses a constraint on the maximum length of a route, the min-max problem does not require a maximum duration constraint, because the objective is to minimize the duration of the longest route.

#### 3. Structural properties of optimal solutions

Dror and Trudeau [8] provided a set of properties for the optimal solution to the (min-sum) Split Delivery Capacitated Vehicle Routing Problem (SDCVRP). In this section, we develop a similar set of properties that provide insights into the structure of an optimal solution to the min-max SDMDVRP with no minimum service time fraction.

**Property 1.** Any min-max SDMDVRP has an optimal solution in which no two routes share more than one customer.

**Proof.** Suppose that, in an optimal solution, routes  $R_1$  and  $R_2$  both service customers  $C_1$  and  $C_2$ , as illustrated in Fig. 1(a). Let  $s_i^{(j)}$  be the service time delivered by route  $R_j$  at customer  $C_i$ , where i, j = 1 or 2. Without loss of generality, assume further that  $s_2^{(1)} \ge s_1^{(2)}$ . We can construct a new solution by transferring the service time spent at  $C_1$  by route  $R_2$  ( $s_1^{(2)}$ ) to route  $R_1$  and, at the same time, transferring the same amount of service time spent at

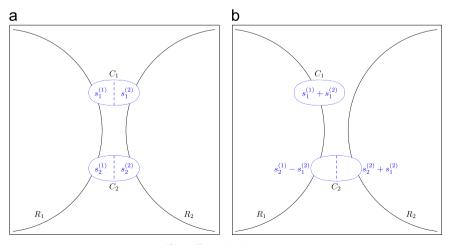


Fig. 1. Illustrating Property 1.

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