Computers & Operations Research 71 (2016) 137–148

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Lagrangian heuristics for the capacitated multi-plant lot sizing problem with multiple periods and items

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ARTICLE INFO

Available online 6 February 2016

Keywords:
Production
Heuristics
Lot sizing problem
Multi-plant

ABSTRACT

Production planning plays an important role in the industrial sector. The focus of this paper is on the lot sizing of those companies composed by multiple plants, each of them with a finite planning horizon divided into periods. All plants produce the same items and have their demands to be met without delay. For producing items, all plants have a single machine with setup times and costs and a limited capacity of production. Transfers of production lots among plants and storage of items are allowed. Even though there are some studies to tackle this problem, to find feasible solutions for the entire set of benchmark instances remains a challenge. This paper introduces novel Lagrangian heuristics that, besides heuristically solving all benchmark instances, significantly outperformed the best heuristic from the literature.

1. Introduction

Production planning has been the subject of several researchers primarily due to the need for addressing the highly competitive productive sector. It is essential for a better use of resources and involves the decision making in a company for manufacturing and delivering products. The long-, medium- and short-term decisions are of utmost importance to optimize the costs related to the production process, from the acquisition of raw materials to final product delivery. Lot sizing, the problem focused in this paper, belongs to the short-term planning [5].

Accordingly, the multi-plant capacitated lot sizing problem (MPCLSP) aims at deciding in a finite planning horizon, the amount, when and what to produce to meet the demands of the plants. In addition, the demands must be satisfied without delay with the lowest cost possible. In particular, this study draws attention to the MPCLSP with multiple items and periods whose plants can produce any item, but have limited time (capacity) for operating the machines in each period. Moreover, in this problem, it is possible to keep inventory and to transfer production lots between plants. One can find in the literature a few studies on the MPCLSP [18,19,16]. Other variants of the lot sizing problem that are highly related to the MPCLSP can be found in [24,12].

Nascimento et al. [16] generated a set of benchmark instances with different sizes and characteristics and put forward a solution method to heuristically solve the MPCLSP. It was not possible to find the optimal solutions for all instances by using the optimization package CPLEX v.7.5 because CPLEX ran out of either time or memory. Therefore, the authors proposed a hybrid metaheuristic that, however, could not find feasible solutions for a number of instances.

The primary goal of this paper is to find sharpened upper and lower bounds for the MPCLSP. In line with this, this paper includes as main contributions novel Lagrangian heuristics that significantly outperformed the state-of-the-art heuristic for the MPCLSP, known as GPheur, in both experiments carried out. Additionally, taking the benchmark instances into account, comparing the achieved results with those obtained by CPLEX v. 12.6 [14] within a time limit of 1800 s, the proposed heuristics, named Lag and LaPRe, were very competitive, with better results for the high setup cost classes of instances. On average, LaPRe was twice better than CPLEX for these instances. Moreover, in an experiment with instances with a higher number of plants and items, Lag and LaPRe performed considerably better than CPLEX v. 12.6 and GPheur.

The remainder of this paper is organized as follows. Section 2 presents the studied integer program. Section 3 loosely details the state-of-the-art heuristic for solving the MPCLSP, and related works. Section 4 shows the proposed hybrid Lagrangian heuristic at length. Section 5 reports an analysis of the experiments carried out for attesting the effectiveness of Lag and LaPRe. To sum up, Section 6 summarizes the primary contributions of this paper and presents some final remarks.

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http://dx.doi.org/10.1016/j.cor.2016.01.019
0305-0548/

2. The multi-plant, multi-item, multi-period capacitated lot sizing problem

This paper approaches the multi-period lot sizing problem with multiple plants, each of them with their amounts of items demanded in the periods (MPCLSP). Sambasivan and Schimidt [18] originally proposed the problem and presented a mathematical formulation, also discussed in [16]. Silva and Toledo [21] recently presented a reformulation of the MPCLSP, presented in formulation (1)–(6). This formulation is strongly related to the model found in [6].

Let m be the number of plants, each of which indexed from 1 to m; a, the number of periods of the planning horizon, each of which indexed from 1 to a; and n, the number of items, each of which indexed from 1 to n. All demands are defined in advance as well as the production capacity of the plants in each period. Additionally, the MPCLSP allows transfers of production lots and the storage of production. For this reason, this study takes into account, for the production planning, the amount of an item i to be produced in a plant-period (j,t) to meet the demand of a plant-period (k,u) for defining the corresponding variable x_{ijtku}.

Moreover, the following parameters must be explicit in the instances of this problem:

- s_{ij}: the setup cost for producing item i at plant j;
- c_{ij}: the unitary production cost of item i at plant j;
- e_{ij}: the unitary inventory cost of item i at plant j;
- r_{jk}: the unitary transportation cost of items from plant j to plant k;
- d_{iku}: the demand of item i at plant k in period u;
- b_{ij}: the processing time of item i at plant j;
- f_{ij}: the setup time for preparing the machine for producing item i at plant j;
- P_{jt}: the production capacity of plant j in period t.

The mathematical formulation is presented next:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{a}\sum_{k=1}^{m}\sum_{u=1}^{a}(\overline{c}_{ijtku}x_{ijtku})+\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{a}(s_{ij}y_{ijt}) \quad (1)$$

subject to

$$\sum_{j=1}^{m}\sum_{t=1}^{u} x_{ijtku} = d_{iku} \quad \forall (i,k,u) \quad (2)$$

$$\sum_{i=1}^{n}\left(f_{ij}y_{ijt}+\sum_{k=1}^{m}\sum_{u=t}^{a} b_{ij}x_{ijtku}\right) \le P_{jt} \quad \forall (j,t) \quad (3)$$

$$x_{ijtku} \le \min\left\{d_{iku};\left\lfloor\frac{P_{jt}-f_{ij}}{b_{ij}}\right\rfloor\right\}y_{ijt} \quad \forall (i,j,t,k,u) \quad (4)$$

$$x_{ijtku} \in \mathbb{N} \quad \forall (i,j,t,k,u) \quad (5)$$

$$y_{ijt} \in \{0,1\} \quad \forall (i,j,t) \quad (6)$$

Let χ_{ijtku} be the costs of inventory and transfers between plants for producing item i at plant j in period t to meet the demand at plant k in period u. Bearing in mind that $r_{jj}=0, \forall j \in \{1,\dots,m\}$, Eq. (7) shows a form to calculate these costs:

$$\chi_{ijtku} = \min_{1 \le v \le m}\{(u-t)e_{iv}+r_{jv}+r_{vk}\} \quad (7)$$

Therefore, for calculating the corresponding costs for producing item i at plant j in period t to satisfy the demand d_{iku}, one may use the following equation:

$$\overline{c}_{ijtku} = \begin{cases} 0, & \text{if } u<t, \\ c_{ij}+\chi_{ijtku}, & \text{otherwise.} \end{cases}$$

According to cost $\overline{c}_{ijtku}$, once the variable x_{ijtku} is positive, item i will be stored from period u to t in the plant with the lowest inventory cost. The objective function, in Eq. (1), aims at finding a solution with the lowest sum of the following costs: production, setup, inventory and transfers between plants. Constraints (2) force the production of the demands of all plants, whilst constraints (3) have as primary goal to keep production within the capacity limit of every plant in any of the periods. Constraints (4) limit the production at plant j in period t to meet the demand of item i of every pair plant-period (k,u) taking the capacity P_{jt} into account. Finally, constraints (5) and (6) define, respectively, the domain of the variables x_{ijtku} to be natural values and y_{ijt} to be binary. Even though this formulation presents an asymptotically significant augment in the number of variables when comparing with the mathematical model introduced in [18], with regard to the instances introduced in [16], it was responsible for tighter linear relaxations [20].

3. Related works

There are a few studies dealing with the MPCLSP in the literature. This specific problem was first investigated in [18]. The case study presented by the authors consists in the production planning of an American manufacturing company of steel rolls with plants positioned in different regions of the country. This type of problem concerns various productive sectors involving large companies, such as beverage corporations, mattress companies and chemical industries [1,2,8]. For heuristically solving the MPCLSP, Sambasivan and Yahya [19] presented a heuristic based on the Lagrangian relaxation of the capacity constraints of the mathematical formulation proposed in [18]. In the cases where the solutions of the approximate relaxed problem were infeasible for the MPCLSP, the authors performed the same strategy used in [18] as an attempt to find feasible solutions.

Later, Nascimento et al. [16] developed a hybrid metaheuristic that significantly outperformed the aforementioned Lagrangian heuristic. This solution method is a Greedy Randomized Search Procedure (GRASP) embedded with the diversification strategy so-called path-relinking [10]. GRASP is a strategy with multiple iterations, each of which with two stages: a semi-greedy construction phase and a local search phase. The construction phase relies on the polynomial algorithm for the uncapacitated lot-sizing problem on parallel facilities [22]. The authors modified such algorithm for being semi-greedy for the uncapacitated multi-plant lot sizing problem. The authors introduced a feasibility strategy that shifts viable production lots among the plants and periods as an attempt to eliminate the capacity violations. If the resulting solution is feasible, then the local search phase starts. Otherwise, the iteration is over. Afterward, the path-relinking starts for then the iteration halts. As both their feasibility phase and local search were employed in the algorithms here proposed, they are thoroughly described in Sections 4.2.1 and 4.2.2.

In addition to the set of instances proposed by Sambasivan and Yahya [19,16] generated larger and harder to solve instances. In spite of its good performance, the GRASP with path-relinking heuristic, named GPheur, could not solve all tested instances. Since then, no efficient strategy has been proposed, the primary reason that supported the proposal of the matheuristics introduced in this paper.