Computers and Mathematics with Applications 61 (2011) 1724–1727

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Semi-foldon fission and fusion in the (2 + 1)-dimensional higher order Broer–Kaup system

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ARTICLE INFO

Article history:
Received 31 October 2010
Accepted 27 January 2011

Keywords:
Broer–Kaup system
Variable separation solution
Soliton fusion and fission

ABSTRACT

We obtain the general variable separation solution of the (2 + 1)-dimensional higher order Broer–Kaup system via the separation variable method, and study semi-foldon fission and fusion based on this variable separation solution.

1. Introduction

Nowadays various methods have been developed to discuss a given nonlinear evolutional equation, such as the darboux method [1], the tanh-function method [2], the Jacobian elliptic function method [3] and the exp-function method [4] and so on. As we all know, interactions between solitons are usually considered to be perfectly elastic in (1 + 1)-dimensional nonlinear models so that no exchange of physical quantities such as energy or momentum is possible between the interacting solitons. However, for some (1 + 1)-dimensional models, two or more solitons may fuse into one soliton at a special time while sometimes one soliton may fission into two or more solitons at other special time [5]. These phenomena are often called the soliton fusion and the soliton fission, respectively. They have been observed in many physical fields like plasma physics, nuclear physics and hydrodynamics and so on [6].

Recently, authors [7,8] discussed some soliton fission and fusion phenomena among the single-valued situations, such as dromions, peakons and compactons. However, in various cases, the real natural phenomena are too intricate to describe only by virtue of the single-valued functions. For instance, in nature, there exist very complicated folded phenomena such as the folded protein [9], folded brain, water surface, and many other kinds of folded biologic systems [10]. Our nature is colorful and may exhibit quite complicated structures such as semi-folded ones [11,12]. Some localized structures such as ocean waves may fold in one direction, say y, and localize in a usual single-valued way in another direction, say x. However, the fusion and fission phenomena among semi-foldons are hardly discussed. In this paper, we will discuss these phenomena in the following higher order Broer–Kaup (HBK) system

$$\begin{aligned} H_t &= -4(H_{xx} + H^3 - 3HH_x + 3H\partial_y^{-1}V_x + 3\partial_y^{-1}(VH)_x)_x, \\ V_t &= -4(V_{xx} + 3HV_x + 3H^2V + 3V\partial_y^{-1}V_x)_x, \end{aligned} \tag{1}$$

which was firstly obtained from the inner parameter dependent symmetry constraints of the KP equation [13]. This Eq. (1) is actually an extension of the so-called Whitham–Broer–Kaup (WBK) system using the Painlevé analysis. The WBK system

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doi:10.1016/j.camwa.2011.01.038

is a valuable model by incorporating or mimicking convective dispersive and viscous effects. In [14], authors obtained many new exact solutions for Eq. (1).

2. Variable separation solution for the (2 + 1)-dimensional HBK system

In order to get the special solution of model (1), we first make a transformation $W = \partial_y^{-1} V_x$, $P = \partial_y^{-1}(VH)_x$. Then, by the cosmetic transformation as rescaling the time t and the dependent variables V, W and P to get rid of the factors "4" and "3", we are able to rewrite (1) in the following potential form:

$$\begin{aligned} &H_t + (H_{xx} + H^3 - 3HH_x + HW + P)_x = 0,\\ &V_t + (V_{xx} + 3HV_x + 3H^2V + VW)_x = 0,\\ &W_y - V_x = 0, \qquad P_y - (VH)_x = 0. \end{aligned} \tag{2}$$

Via the standard truncated Painlevé expansion [9], we have a special Painlevé–Bäcklund transformation for these differentiable functions H, V, W and P in (2)

$$H = (\ln f)_x + H_0, V = (\ln f)_{xy} + V_0, W = (\ln f)_{xx} + W_0, P = (\ln f)_x(\ln f)_{xx} + P_0, \tag{3}$$

where $f = f(x, y, t)$ is an arbitrary differentiable function of variables $\{x, y, t\}$ to be determined, and H_0, V_0, W_0, P_0 are arbitrary seed solutions satisfying the HBK system. In usual cases, by choosing some special trivial solutions, we can directly obtain the seed solutions $H_0 = V_0 = 0$, $W_0 = W_0(x, t)$, $P_0 = P_0(x, t)$.

Similarly to the procedure in [14], substituting (3) with the seed solutions into (2) yields

$$f_t + f_{xxx} + W_0 f_x + P_0 f = 0. \tag{4}$$

Concerning the linear equation (4) of the original system, we can construct many types of special solutions. One of them is $f = p(x, t) + q(y)$, thus we obtain the variable separation solution of the (2 + 1)-dimensional HBK system

$$H = \frac{p_x}{p+q}, \tag{5}$$

$$V = -\frac{p_x q_y}{(p+q)^2}. \tag{6}$$

3. Semi-foldon fission and fusion

Rich localized coherent structures such as non-propagating solitons, dromions, peakons, compactons, foldons, et al. are neglected here since they have been reported in [15–18]. Here we pay our attention to these intriguing fission and fusion phenomena for the multi-valued semi-foldons.

4. Fission phenomenon of single bell-like semi-foldon

If two arbitrary functions p and q in (6) are considered as

$$p = -12 - [\exp(5x + 6t) - 0.5\exp(2x + 3t) + 0.8\exp(2x + 4t)][1 - \exp(2x + 3t)]^{-2}, \tag{7}$$

$$q_y = -\sec h^2(\zeta), \qquad y = \zeta - 1.3\tan h(\zeta), \qquad q = \int^{\zeta} q_y y_\zeta \, d\zeta, \tag{8}$$

then one can obtain a new kind of fission solitary wave solution. In Fig. 1(a), the bell-like semi-foldon folds as the loop soliton in the y-direction and localizes as the bell-like soliton in the x-direction. From Fig. 1(a)–(d), one can clearly see that the single bell-like semi-foldon fissions into three bell-like semi-foldons (two semi-foldons and one anti-semi-foldon) with different amplitudes. It is interesting to mention that these bell-like semi-foldons exhibit different evolutional behaviors, i.e., the smaller semi-foldon is almost static in position $x = 2, y = 0$, and the bigger semi-foldon and the anti-semi-foldon travel along negative x-axis. They are stable and do not undergo additional fission at least not if running the program for longer period of $t = 65$.

5. Fusion phenomenon between bell-like and peakon-like semi-foldons

Besides these fusion phenomena between localized coherent structures discussed in [3,4], we also find a novel fusion phenomenon between multi-valued bell-like and peakon-like semi-foldons. If p possesses the following form

$$p = 22 + \exp(x + t) + \begin{cases} 4\exp(2x - 2t) & x - t \le 0,\\ -4\exp(2t - 2x) & x - t > 0, \end{cases} \tag{9}$$

and q is taken as (8), one can observe the intriguing fusion phenomenon, plotted in Fig. 2. The right semi-foldon folds as the loop soliton in the y-direction and localizes as the peakon in the$x$-direction, so we call it a peakon-like semi-foldon.