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On soft topological spaces

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ABSTRACT

In the present paper we introduce soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms are introduced and their basic properties are investigated. It is shown that a soft topological space gives a parametrized family of topological spaces. Furthermore, with the help of an example it is established that the converse does not hold. The soft subspaces of a soft topological space are defined and inherent concepts as well as the characterization of soft open and soft closed sets in soft subspaces are investigated. Finally, soft T_i-*spaces* and notions of soft normal and soft regular spaces are discussed in detail. A sufficient condition for a soft topological space to be a soft T_1-*space* is also presented.

1. Introduction

The real world is too complex for our immediate and direct understanding. We create "models" of reality that are simplifications of aspects of the real word. Unfortunately these mathematical models are too complicated and we cannot find the exact solutions. The uncertainty of data while modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical sciences and many other diverse fields makes it unsuccessful to use the traditional classical methods. These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world or to the limitations of the means used to measure objects. For example, vagueness or uncertainty in the boundary between states or between urban and rural areas or the exact growth rate of population in a country's rural area or making decisions in a machine based environment using database information. Thus classical set theory, which is based on the crisp and exact case may not be fully suitable for handling such problems of uncertainty.

There are several theories, for example, theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2], theory of vague sets, theory of interval mathematics [3,4] and theory of rough sets [5]. These can be considered as tools for dealing with uncertainties but all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov in [6]. He initiated the concept of soft set theory as a new mathematical tool which is free from the problems mentioned above. In his paper [6], he presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly.

Maji et al. [7,8] presented an application of soft sets in decision making problems that is based on the reduction of parameters to keep the optimal choice objects. Chen [9] presented a new definition of soft set parametrization reduction

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and a comparison of it with attribute reduction in rough set theory. Pei and Miao [10] showed that soft sets are a class of special information systems. Kong et al. [11] introduced the notion of normal parameter reduction of soft sets and its use to investigate the problem of sub-optimal choice and added a parameter set in soft sets. Zou and Xiao [12] discussed the soft data analysis approach. The application of soft set theory in algebraic structures was introduced by Aktaş and Çağman [13]. They discussed the notion of soft groups and derived some basic properties. They also showed that soft groups extended the concept of fuzzy groups. Jun [14,15] investigated soft BCK/BCI-algebras and its application in ideal theory. Feng et al. [16] worked on soft semirings, soft ideals and idealistic soft semirings. Ali et al. [17] and Shabir and Irfan Ali [17,18] studied soft semigroups and soft ideals over a semigroup which characterize generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup.

The main purpose of this paper is to introduce soft topological spaces which are defined over an initial universe with a fixed set of parameters. First we give some basic ideas about soft sets and the results already studied. Then we discuss some basic properties of soft topological spaces and define soft open and closed sets. The soft closure of a soft set is defined which is, in fact, a generalization of closure of a set in a broader sense. The newly introduced concept of parameters comes into play with the collection of parametrized topologies on the initial universe. Corresponding to each parameter, we get a topological space and this makes the involvement of parameters more significant. We can say that a soft topological space gives a parametrized family of topologies on the initial universe but the converse is not true i.e. we cannot construct a soft topological space if we are given some topologies for each parameter and this is shown in detail with the help of examples in this paper. Consequently we can say that the soft topological spaces are more comprehensive and generalized than the classical topological spaces. During the process of theory development we also see that the properties of parametrized topologies correspond to that of soft spaces in some particular situations. Finally soft separation axioms for soft topological spaces are defined and some interesting results are derived which may be of value for further research. Although most of the results, discussed in this paper, are very basic and provide an introductory platform but potentially useful research in theoretical as well as applicable directions can be made. One possible inspirational thought is of modeling topological relations between spatial objects. Mathematically, point set topology can be applied as a fundamental tool for modeling crisp spatial objects in GIS. Spatial and non-spatial information about entities is represented by attributes. The models developed so far, do not consider attribute aspects of spatial objects so the soft topology may be a handy tool for this purpose. The resemblance of soft sets with that of a query from databases may be a useful side for the modeling process.

2. Soft sets

Definition 1 ([6]). Let U be an initial universe and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon-$approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2 ([8]). For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

(1) $A \subseteq B$ and
(2) for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations.

We write $(F, A)\tilde{\subset}(G, B)$.
(F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\tilde{\supset}(G, B)$.

Definition 3 ([8]). Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 4 ([8]). Let $E = \{e_1, e_2, \ldots, e_n\}$ be a set of parameters. The *NOT* set of E denoted by $\rceil E$ is defined by $\rceil E = \{\rceil e_1, \rceil e_2, \ldots, \rceil e_n\}$ where, $\rceil e_i = not\ e_i$ for all i.

The following results are obvious.

Proposition 1 ([8]).

(1) $\rceil(\rceil A) = A$;
(2) $\rceil(A \cup B) = \rceil A \cup \rceil B$;
(3) $\rceil(A \cap B) = \rceil A \cap \rceil B$.

Definition 5 ([8]). The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$ where, $F^c : \rceil A \to \mathcal{P}(U)$ is a mapping given by $F^c(\alpha) = U \setminus F(\rceil\alpha)$, for all $\alpha \in \rceil A$.

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 6 ([8]). A soft set (F, A) over U is said to be a *NULL* soft set denoted by Φ if for all $\varepsilon \in A$, $F(\varepsilon) = \emptyset$ (null set).