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Solving large-scale multidimensional knapsack problems with a new binary harmony search algorithm

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ABSTRACT

Harmony search (HS) is a meta-heuristic method that has been applied widely to continuous optimization problems. In this study, a new binary coded version of HS, named NBHS, is developed for solving large-scale multidimensional knapsack problem (MKP). In the proposed method, focus is given to the probability distribution rather than the exact value of each decision variable and the concept of mean harmony is introduced in the memory consideration. Unlike the existing HS variants which require specifications of parameters such as the pitch adjustment rate and step bandwidth, an ingenious pitch adjustment scheme without parameter specification is executed in the proposed HS according to the difference between two randomly selected harmonies stored in the harmony memory to generate a new candidate harmony. Moreover, to guarantee the availability of harmonies in the harmony memory, a simple but effective repair operator derived from specific heuristic knowledge of the MKP is embedded in the proposed method. Finally, extensive numerical simulations are conducted on two sets of large-scale benchmark problems, and the results reveal that the proposed method is robust and effective for solving the multidimensional knapsack problems with large dimension sizes.

1. Introduction

Combinatorial optimization plays a very important role in operational research, discrete mathematics and computer science. The 0–1 knapsack problem is a well-known type of combinatorial optimization problem and it can represent the factory location problem, the production scheduling problem, the assignment problem, the reliability problem, etc. It derives its name from the problem faced by someone who is constrained by a special knapsack with many limitations but wish to pack it with the most valuable items.

The multidimensional knapsack problem (commonly known as the MKP) is generalized from the standard 0–1 knapsack problem. The MKP is a subset selection process of given items with specific profit and resource occupation to fulfill a knapsack without exceeding multidimensional resource capacities. Those items are appropriately chosen to make the cumulative profit packed in the knapsack as large as possible.

Mathematically, an m-dimensional knapsack problem with n items in its standard form can be described as

$$\text{Max} \quad f(\mathbf{x}) = \sum_{i=1}^{n} p_i \cdot x_i$$

$$\text{s.t.} \quad \sum_{i=1}^{n} r_{i,j} \cdot x_i \le R_{max,j}, \quad j = 1, 2, \ldots, m$$

$$x_i \in \{0, 1\}, \quad i = 1, 2, \ldots n \tag{1}$$

where each item i ($i=1,2,\ldots,n$) has a profit value p_i and consumes an amount $r_{i,j}$ for the j-th ($j=1,2,\ldots,m$) resource. The objective is to maximize the total profits of all the items in a subset while the j-th resource occupation of the subset must be less than its corresponding resource capacity $R_{max,j}$. The value of x_i representing the state of the item i in the knapsack is restricted to be either 0 or 1. If the item i is put into the knapsack, the value of x_i is set to be 1, otherwise, 0. Each item may be chosen at most once and cannot be placed in the knapsack partly. Without loss of generality, it can be assumed that the above parameters in the MKP are non-negative integers and the following constraints, as defined by Eq. (2), must be satisfied:

$$r_{i,j} \le R_{max,j}, \quad \sum_{i=1}^{n} r_{i,j} \ge R_{max,j}, \quad j = 1, 2, \ldots, m \tag{2}$$

Besides the number of constraints and the number of variables, there is another important parameter in the MKP which determines the resource capacities $R_{max,j}$. The slackness ratio S_j ($j=1,2,\ldots,m$) introduced by Zanakis [1] defined as Eq. (3) is adopted here to generate different levels of the resource

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capacities:

$$S_j = \frac{R_{max,j}}{\sum_{i=1}^{n} r_{i,j}}, \quad j = 1, 2, \ldots, m \tag{3}$$

Since the MKP is a well-known NP-hard problem which arises various engineering fields, many efficient, exact and approximate algorithms have been developed for obtaining optimal or near-optimal solutions. Early exact methods including dynamic programming [2], hybrid dynamic programming method [3] and branch and bound algorithm [4,5] can be applied only to some small-scaled instances in an acceptable computation time. They become insufficient in solving large-scale problems because the computation complexity is rather high and the resulting computational effort and memory requirement can be tremendous. With the development of computational intelligence in recently years, more and more researchers focus on heuristic and meta-heuristic search methods for finding a high quality sub-optimal solution instead of the optimal solution. However, the meta-heuristics are not guaranteed to find the optimum. Relevant algorithms include genetic algorithm [6], simulated annealing [7], tabu search [8], ant colony optimization [9], particle swarm optimization [10], and so on. Most of the above algorithms require solving the linear programming relaxation of the original MKP which becomes difficult even infeasible as the dimension increased. Like the exact methods, these methods fail to be effective and efficient for solving large-scale MKPs. Very recently, an estimation of distribution algorithm based hybrid algorithm named HEDA has been proposed by Wang et al. [11] to cope with large-scale problems by using a new repair operator.

Among numerous heuristic and meta-heuristic methods proposed in the last two decades, the harmony search (HS) algorithm, a simple but powerful stochastic search technique for solving global optimization problems proposed by Geem et al. [12], is worth mentioning. Inspired by the musician attuning such as during rock music improvisation, the harmony search algorithm imitates the improvisation processes to find a perfect pleasing harmony from an aesthetic point of view which is similar to the seeking process in optimization to get a global optimal solution evaluated by an objective function. For an optimization problem, a musical harmony in the HS algorithm can be seen as the variable vector and the best harmony achieved in the end is analogous to the global optimum.

Several characteristics and advantages of HS have been discussed in [13–15]. The key differences between HS and other meta-heuristics, such as genetic algorithm (GA), particle swarm optimization (PSO) and differential evolution (DE), include: (1) HS enables each existing harmony vector to participate in the generation of a new solution vector. This increases the flexibility and helps to produce better solutions. While GA only considers two selected individuals as the parent vectors, PSO adjusts the trajectory of each particle simply toward its personal best location and the fittest position of the entire swarm found so far, and DE combines just three distinct individuals to form an offspring vector. (2) HS determines the value of each decision variable via improvisation independently. This is in contrast to PSO and DE in which a vector is adjusted according to a fixed rule. (3) A single solution vector is obtained after considering all of the existing vectors in HS, whereas GA, PSO and DE allow the simultaneous production of multiple solutions. (4) The newly generated solution is checked with the worst harmony among the existing ones in the harmony memory. Unlike HS, the offspring solutions compete with their corresponding parent individuals in many other meta-heuristics including GA, PSO and DE.

Owing to its advantage that there are fewer mathematical parameters to adapt than other meta-heuristic algorithms, the HS algorithm has been successfully applied to complex real-world optimization problems including parameter identification [16,17], structural optimization [18,19], reliability problems [20,21], dispatch problems [22,23], scheduling problem [24], unit commitment problems [25], classification problems [26], network reconfiguration [27], etc.

Most of the existing variants of HS mentioned above are focusing on optimization problems in continuous space. Until now, only a few papers concentrate on discrete problems and binary problems. Geem [28] introduced a binary coded HS without pitch adjustment operator to tackle a discrete problem, i.e., the water pump switching problem. Greblicki and Kotowski [29] analyzed the defect on the search ability degraded by discarding of pitch adjustment operator from the theoretical and experimental results. To circumvent the above weakness, a new pitch adjustment operation was presented by Wang at al. [30] and a novel discrete binary HS algorithm was developed to solve the discrete problems effectively. More recently, a scalable adaptive strategy was developed by Wang at al. [31] in an improved adaptive binary harmony search (ABHS) algorithm. Experimental results on the benchmark functions and 0–1 knapsack problems have demonstrated that it is beneficial to enhance the search ability and robustness in order to solve the binary-coded problems more effectively. Owing to its outstanding performance, ABHS was extended to search the optimal parameters of the fuzzy controller for improving the control performance with the guaranteed stability afforded by LFC [32].

Unlike the previous binary coded HS variants, several real-coded methods have also devoted to solve the discrete problems through specific implementation of transformation from real variables to actual discrete decision values. The most direct and commonly used strategy is to replace a real number with the nearest integer which corresponds to a permissible discrete decision value. Based on this observation, a novel global harmony search algorithm (NGHS) derived from the swarm intelligence of particle swarm was developed by Zou et al. [33] for solving the 0–1 knapsack problems. NGHS introduced a new position updating scheme and genetic mutation strategy to replace the harmony memory consideration and pitch adjusting in the classical HS. Similar to NGHS, a social harmony search algorithm model was presented by Kaveh and Ahangaran [34] for the cost optimization of composite floor system with discrete variables. Besides the conversion of actual discrete decision values, special improvisation operators are also designed to enable the search executed directly in the discrete domain. For example, the new pitch adjustment with neighboring values introduced by Lee et al. [35] helps HS to optimize the structures with discrete-sized members. With the assistance of job-permutation-based representation, Gao et al. [36] employed a novel pitch adjustment rule to produce a feasible solution such that HS is effective for solving the no-wait flow shop scheduling problems with total flow time criteria. Note that a novel partial stochastic derivative [37] was defined for discrete-valued functions and embedded in harmony search to find the optimal solutions for various science and engineering problems. However, the performances of the existing discrete HS variants are not satisfactory. In other words, the research on HS algorithms for discrete problems is still at its infancy.

To the best of the authors' knowledge, there is only one variant of HS method for solving the MKP. That is, quantum inspired harmony search algorithm (QIHS) proposed by Layeb [38] which allows successful application of some quantum inspired operators such as measurement and interference in basic harmony search algorithm. Although Layeb demonstrated the feasibility and the effectiveness of the QIHS based on experimental studies with different types of instances and problem sizes, it is not evident enough to claim that the proposed QIHS algorithm performs better than others mentioned in the paper. The numerical experimentation is too limited