Computers & Operations Research 63 (2015) 136–148

Contents lists available at ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Hybrid methods for lot sizing on parallel machines

Diego Jacinto Fiorotto [a], Silvio Alexandre de Araujo [a,*], Raf Jans [b]

[a] Departamento de Matemática Aplicada, Univ. Estadual Paulista, 15054-000 São José do Rio Preto, SP, Brazil
[b] HEC Montréal and CIRRELT, Canada H3T 2A7 QC, Canada

ARTICLE INFO

Available online 1 May 2015

Keywords:
Lot sizing
Parallel machines
Reformulation
Hybrid methods
Dantzig–Wolfe decomposition
Lagrangian relaxation

ABSTRACT

We consider the capacitated lot sizing problem with multiple items, setup time and unrelated parallel machines, and apply Dantzig–Wolfe decomposition to a strong reformulation of the problem. Unlike in the traditional approach where the linking constraints are the capacity constraints, we use the flow constraints, i.e. the demand constraints, as linking constraints. The aim of this approach is to obtain high quality lower bounds. We solve the master problem applying two solution methods that combine Lagrangian relaxation and Dantzig–Wolfe decomposition in a hybrid form. A primal heuristic, based on transfers of production quantities, is used to generate feasible solutions. Computational experiments using data sets from the literature are presented and show that the hybrid methods produce lower bounds of excellent quality and competitive upper bounds, when compared with the bounds produced by other methods from the literature and by a high-performance MIP software.

1. Introduction

This paper deals with a lot sizing problem that consists basically of determining the size of production lots, i.e. the amounts of each item to be produced, in each of the periods in the planning horizon in a way that minimizes total production costs, respects resource availability and meets the demand of the items. The problem studied here involves the production of multiple items in a single stage. The production sector consists of unrelated parallel machines with limited capacity. The items can be produced on any of the machines and, several different items can be produced on the same machine in the same time period (large bucket model). At the start of the production of each type of item, there is a setup time and a setup cost for the machine being used and, the setup is sequence-independent.

The paper has the following contributions. First, we propose a way to obtain lower bounds that are stronger than the ones obtained by the traditional per-item Dantzig–Wolfe decomposition. Second, we extend two hybrid algorithms that combine Lagrangian relaxation and Dantzig–Wolfe decomposition and apply them to obtain the stronger lower bounds for the problem with unrelated parallel machines. Third, we improve the Lagrangian heuristic proposed by Fiorotto and de Araujo [20] to obtain better upper bounds. Finally, computational experiments are performed to show the quality of the upper bounds and lower bounds compared to other methods from the literature. A comparison shows that the new hybrid method together with the improved heuristic provides generally better gaps.

The paper is organized as follows. In Section 2, we provide a literature review on lot sizing problems on parallel machines and Dantzig–Wolfe decomposition. In Section 3, the classical formulation of the problem is presented along with the proposed reformulation. In Section 4, we present the techniques of Lagrangian relaxation and Dantzig–Wolfe decomposition applied to the lot sizing problem on parallel machines. Section 5 describes the proposed algorithms to calculate these lower bounds. In Section 6, the Lagrangian heuristic used is summarized and, in Section 7, the computational results are presented. Finally in Section 8, we present our conclusions.

2. Literature review

In this section, we will first discuss papers related to lot sizing problems on parallel machines and subsequently we discuss relevant papers involving Dantzig–Wolfe decomposition and column generation applied to lot sizing problems.

2.1. Literature review on parallel machine lot sizing

In practical production planning problems, parallel machines often need to be taken into account. Areas of production that consider parallel machines are the pharmaceutical industry [16],

* Corresponding author
E-mail addresses: diego_fiorotto@homail.com (D.J. Fiorotto), saraujo@ibilce.unesp.br (S.A. de Araujo), raf.jans@hec.ca (R. Jans).

http://dx.doi.org/10.1016/j.cor.2015.04.015
0305-0548/

plastic sheet production [35], tile production [15], the tire industry [29], bottling of liquids and others [10] and packaging [34].

Considering the problem with identical parallel machines, Lasdon and Terjung [32] propose a heuristic for a lot sizing and scheduling problem with no machine setup time. Carreno [10] proposes a heuristic for the Economic Lot Scheduling Problem (ELSP), i.e. with a constant demand rate, with setup times for parallel machines and solves problems with 100 items and 10 machines in fast computational times. Jans [27] proposes new constraints to break the symmetry that is present due to the identical machines and tests his approach using a network reformulation for the problem. Tempelmeier and Buschkuhl [43] consider the multi-stage problem with setup carry-over (a setup is maintained between adjacent periods) and develop a Lagrangian heuristic.

For the unrelated parallel machines case, Toledo and Armentano [44] relax the capacity constraints and propose a Lagrangian heuristic to solve the problem. An initial solution is obtained by minimizing the Lagrangian problem, which is normally infeasible. In an attempt to make it feasible production is shifted between periods and machines, moving the production that exceeds the capacity and looking for feasible solutions that minimize the cost. Fiorotto and de Araujo [20] study the same problem. The authors use a strong reformulation of the problem and instead of the capacity constraints, they relax the demand constraints using Lagrangian relaxation. They also propose a heuristic to find feasible solutions and compare their results with Toledo and Armentano [44].

Multi-stage problems with unrelated parallel machines were studied in Ozdamar and Birbil [38], who present a generic model in which the multi-stage case can be considered. Three hybrid heuristics are developed, in which a tabu search algorithm is used to make the problem feasible and improve the solutions. Stadtler [42] and Helber and Sahling [23] also analyze the multi-stage problem. Stadtler [42] proposes a period decomposition heuristic and, to solve each subproblem, a reformulation based on the facility location problem is used. Helber and Sahling [23] propose a fix-and-optimize approach and obtain better results than those obtained by Stadtler [42].

Some research in the literature deals with the problem of parallel machines and sequence-dependent setup costs and times. Some of the research papers consider small bucket models, where only one item can be produced per machine per period. Salomon et al. [40] study the Discrete Lot Sizing and Scheduling Problem (DLSP) with parallel machines, and analyze the complexity for the cases of identical and non-identical machines. The authors also present some solution algorithms. Kang et al. [30] propose a method based on column generation and branch-and-bound. Belvaux and Wolsey [5] describe a generic model and an optimization system that is capable of solving a wide range of lot sizing problems including special cases with different items and parallel machines. Meyr [36] presents a general model that consists of an extension of the General Lot Sizing and Scheduling Problem (GLSP) model for the case in which both setup cost and time are sequence-dependent. Fandel and Stammen-Hegener [19] also present a model based on the GLSP model and consider the multi-stage case. Marinelli [34] proposes a solution approach for a real capacitated lot sizing and scheduling problem with parallel machines and shared buffers, arising in a packaging company producing yoghurt. Finally, Meyr and Mann [37] propose a heuristic for the lot sizing and scheduling problem on parallel machines. Different decomposition approaches are proposed and compared with results from the literature.

2.2. Literature review on Dantzig–Wolfe decomposition

Dantzig–Wolfe decomposition and column generation have been used to find good quality lower bounds for lot sizing problems. Before the seminal paper of Dantzig and Wolfe [13], Manne [33] had already implicitly applied the ideas of decomposition for the lot sizing problem with dynamic demand considering several items and capacity constraints. Manne proposed a linear programming model that explicitly considers all possible production schedules. Lambrecht and Vanderveken [31], Bitran and Matsuo [7] and Degraeve and Jans [14] further discuss the formulation proposed by Manne [33].

Degraeve and Jans [14] show that the decomposition proposed by Manne, while valid to calculate a strong lower bound, has a structural deficiency when it aims to solve the problem with integrality constraints. The set of feasible solutions for Manne's formulation with integrality constraints, is only a subset of feasible solutions for the original integer problem. The main reason for this deficiency is that the solution for the subproblems, i.e. a new column, consists of both setup and production variables and in Manne's formulation the decisions of the setup automatically determine the production quantity decisions according to the Wagner–Whitin property. However, it is very likely that the optimal solution for the capacitated problem will not have this property.

Dzielinski and Gomory [17] use column generation to handle the formulation with the large number of variables proposed by Manne [33]. Indeed, Manne's formulation is the full master problem obtained when one applies Dantzig–Wolfe decomposition [13] to a formulation with a smaller number of variables. Dzielinski and Gomory [17] also note that the subproblems that must be solved to generate columns are equivalent to the problem studied by Wagner and Whitin [46].

Lasdon and Terjung [32] develop a column generation approach to handle large problems. Algorithms of this type are also addressed by Bahl [3], Cattrysse et al. [11], Salomon et al. [41] and Huisman et al. [26].

Hindi [24] presents a heuristic including variable redefinition and column generation. To calculate the upper bound he solves a minimum cost flow problem. Hindi [25] combines the ideas of linear relaxation, column generation, minimum cost flow network and Tabu search in a hybrid algorithm. Haase [22] also solves the lot sizing problem by column generation and finds improved lower bounds.

Considering that both Lagrangian relaxation and Dantzig–Wolfe decomposition have advantages and disadvantages, Huisman et al. [26] discuss two different ways to combine these two methods in hybrid algorithms to solve the linear relaxation of the master problem. In the first approach they apply Lagrangian relaxation to solve the master problem, i.e., no simplex method is used. In the second approach, they use the simplex method to generate the optimal dual variables of the master problem and the Lagrangian relaxation approach to generate columns. In this latter approach the Lagrangian relaxation is applied to the compact formulation. The ideas are illustrated using the lot sizing problem.

Pimentel et al. [39] consider the lot sizing problem with setup time and apply the Dantzig–Wolfe decomposition to the classical formulation in two different ways: item decomposition and period decomposition. Furthermore, a third decomposition is presented which applies decomposition per item and period simultaneously. The authors conclude that this last approach provides better lower bounds than those obtained by the other decompositions. They implemented three branch-and-price algorithms to solve the three decomposition models.

de Araujo et al. [2] present a transformed reformulation and valid inequalities that speed up column generation and Lagrangian relaxation for the capacitated lot sizing problem with setup times (CLST) and show theoretically how both ideas are related to dual space reduction techniques. Finally, the authors propose a combination of the two methods proposed by Huisman et al. [26]. This approach obtains good computational results and avoids the need of a linear programming optimization package.