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# Mathematical models and algorithms for a high school timetabling problem

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ABSTRACT

This paper investigates a high school timetabling problem in a case study related to Kuwait's public educational system, which is concerned with assigning teachers to classes and time-slots. Because a direct solution to an initially formulated comprehensive mixed-integer programming model for generating weekly teacher schedules was found to be untenable for practical-sized realistic test instances, we propose in this paper two decomposition approaches to the underlying problem. A two-stage modeling solution approach is presented first, where the initial stage determines weekly time-slots for the classes, based on which, the second stage then assigns teachers to classes. Instead of generating weekly schedules within the model itself, we propose another mixed-integer programming formulation that selects valid combinations of weekly schedules from the set of all feasible schedules, and we design a column generation solution framework to exploit its inherent special structure. Computational results are presented for the proposed solution approaches using several real as well as simulated realistic test problems pertaining to high schools in Kuwait.



## 1. Introduction

This section presents an overview of the studied problem in Section 1.1, and then reviews the related research available in the open literature in Section 1.2. Finally, Section 1.3 discusses the contribution and organization of this paper.

### 1.1. Problem specifics

In this research effort, we consider a challenging high school timetabling problem that is ubiquitous in many countries worldwide. The specific problem investigated in this paper is concerned with generating class-teacher timetables for high schools in Kuwait's public educational system. In this context, there are three grade levels: tenth, eleventh, and twelfth. The class requirements for the tenth grade (*i.e.*, subjects and hours for each subject) are fixed for all students and are known in advance. For the eleventh grade, the students have the choice of specializing in Science or Liberal Arts, where again, the class requirements are fixed and known. A student who has completed the eleventh grade in Science proceeds to the twelfth grade with the same specialization, and likewise for a student who has completed the eleventh grade in Liberal Arts. We aim to find weekly class-teacher schedules without violating any of the problem's (hard) constraints that include: (a) avoiding the assignment of more than one teacher to a class during any given time-slot; (b) avoiding the assignment of a teacher to more than one class during any given time-slot, and (c) not exceeding the daily and weekly teaching loads of teachers.

### 1.2. Related literature

Academic institutions worldwide face complex timetabling problems related to scheduling courses, exams, faculty members, teaching assistants, and the like. The generation of effective and robust timetabling tools for concerned decision makers requires efficient quantitative and algorithmic approaches due to the highly combinatorial nature of such problems, especially for relatively large institutions. Timetabling problems that incorporate multiple critical features are typically challenging and theoretically NP-hard [19,23,29,31,33]. Several surveys related to academic timetabling problems have been presented in the literature; see for example, [20,23,48,57,61,68]. Also, McCollum [50] and McCollum et al. [51] have provided insightful discussions that focus on bridging the gap between research and practice in this area. In this context, increasing attention has been dedicated to timetabling

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problems in general, as evidenced from: (a) the series of biannual international conferences on the Practice and Theory of Automated Timetabling (PATAT); (b) the establishment of prominent research groups, such as WATT and the EURO Working Group on Automated Timetabling, which were established to "discuss, promote, and perform research related to automated timetabling issues and methods"; and (c) the vast number of graduate theses and dissertations related to timetabling problems, e.g., [38,41,44,47,49,52,63,79].

High school timetabling problems vary depending on the country and type of educational system [61]. Country-specific literature includes that related to Australian schools [52], Dutch schools [28,38,41,69,79], Finnish schools [49], German schools [39,40], Greek schools [14,15,17,56], Hong Kong schools [43], Italian schools [7], South African schools [62], and Spanish schools [6,8]. This body of research has produced valuable theoretical, conceptual, and practical contributions. Moreover, to facilitate comparative studies, a *School Benchmarking Project* was launched in 2007 (see Post et al. [58,59,60]), where a general file format was introduced based on typical common features, and where researchers from all over the world have contributed instances pertaining to their local schools. The website dedicated for this project (http://opt-kd.cse.dmu.ac.uk/www/information.php) also contains insightful and important information about high school timetabling such as terminology, references, history, links, algorithms, and most importantly, a data repository. This project is still active and the continued contributions of researchers in this domain are valuable.

Several algorithmic approaches have been used in the literature to tackle high school and related timetabling problems, including ant colony algorithms [32,70], constraint programming [36,46,77], evolutionary algorithms [14], genetic algorithms [15,34,62,63,76], graph theory-based procedures [16], mixed-integer programming methods [9,17,18,25,26,45,56,65,75], local search schemes [10,67], metaheuristics [24,27], neural network algorithms [21,22,35], programmed search techniques [37], simulated annealing [2,3,4,74,81], and tabu search heuristics [1,5,7,8,11,39,64,66]. A number of high school related survey papers are also available in the literature-for example, Junginger [40] has surveyed high schools in Germany that use computer software for their timetabling, and Kwok et al. [43] have surveyed approaches that pertain to the timetabling process for high schools in Hong Kong. Also, Schaerf [68] has presented a general survey on automated timetabling, along with a special section devoted to high school timetabling. Given these extensive discussions, we focus in the remainder of this section on some relatively recent work related to high school timetabling.

Wood and Whitaker [80] formulated a nonlinear goal programing model for a secondary school timetabling problem, where students freely choose their courses of study from a list of subjects. Similar to the problem dealt with in the present paper, Birbas et al. [17] considered a timetabling problem faced by high schools in Greece. The authors presented an integer programming model for scheduling a large number of classes, teachers, courses, and classrooms with respect to a number of time-slots. Computational results for a typical Greek high school were presented to demonstrate the effectiveness of the developed model in satisfying both hard and soft problem constraints. Valousix and Housos [77] also addressed this same type of problem and solved it using a constraint programming (CP) approach. Local search techniques were used to assist the CP search process by effectively reducing the solution search space, and specialized algorithms were developed to compute lower bounds for providing performance guarantees. Computational results for a number of specific test problem instances were presented. A column generation approach for the problem investigated in [17] was proposed by Papoutsis et al. [56] based on a 0–1 integer programming model in which each column represents a valid schedule of a given teacher. The authors solved the linear programming relaxation of the resulting model by sequentially generating promising new columns via a network-based heuristic, and accordingly designed an overall heuristic procedure to solve the original discrete problem. Birbas et al. [18] proposed an integer programming approach for a high school timetabling problem related to the Hellenic secondary educational system in Greece. A shift assignment problem that optimally assigns shifts to teachers was solved first, and then tasks within shifts were assigned to the corresponding teachers. Using this combined approach, the authors were able to generate conflict free, complete, fully compact, and well balanced timetables for students. Daskalaki et al. [25] and Daskalaki and Birbas [26] developed mixed-integer programming approaches for a timetabling problem related to Greek universities. In the latter paper, a two-stage relaxation procedure was designed, where the computationally heaviest constraints were initially relaxed in the first stage, and were resurrected in the second stage for generating timetables on a day-by-day basis in order to construct a good quality solution for the original problem. Tripathy [75] formulated an MIP model for a timetabling problem and presented a solution method based on coordinating Lagrangean relaxation and subgradient optimization with a branch-and-bound procedure using special ordered sets of variables.

Carrasco and Pato [21,22] explored the application of a Potts neural network-based metaheuristic to a class-teacher timetabling problem. Avella et al. [10] proposed a local search algorithm for a high school timetabling problem taking into account both hard and soft constraints. The authors employed simulated annealing with a very large-scale neighborhood search approach, where this neighborhood was explored by solving an integer programming problem. Haan et al. [28] presented a three-step tabu search approach for generating a timetable for a Dutch high school. Beligianni et al. [15] developed a genetic algorithmic procedure for high schools in Greece and tested it using exhaustive computational experiments along with real-world data. Moura et al. [53] examined a constrained high school timetabling problem and applied a Greedy Randomized Adaptive Search Procedure (GRASP) heuristic, combined with a path-relinking improvement step. Zhang et al. [81] used a simulated annealing-based heuristic with a newly-designed neighborhood structure to approximately solve a high school timetabling problem, where the procedure performs a sequence of swaps between pairs of time-slots, instead of between pairs of assignments as in traditional simulated annealing approaches for this problem. Ramirez [63] used genetic algorithms to solve two high school timetabling problems, with and without a fixed master schedule, while employing smart operators during the mutation process. Kingston [42] presented several algorithms for the assignment of teachers and rooms to meetings, assuming that the meeting time-slots are known. These algorithms were tested on real-world data, the best of which is currently implemented within a free public software for high school timetabling. Raghavjee and NePillay [62] designed a two-phase genetic algorithmic approach for solving a high school timetabling problem in South Africa.

Dorneles et al. [30] proposed a novel framework for solving a variant of the high school timetabling problem for the Brazilian schools. The authors formulated a mixed- integer programming model and developed a fix-and-optimize heuristic approach combined with a variable neighborhood descent method to solve the developed model. Odeniyi et al.. [55] investigated the school timetabling problem of the Fakunle Comprehensive High School (FCHS) in Osogbo, Nigeria. The authors utilized a modified simulated annealing (MSA) approach for solving the underlying timetabling problem. Sørensen and Dahms [71] adopted a two-stage decomposition approach of an IP model formulated for a high school timetabling problem in Denmark. This particular timetabling problem aims to assign each lecture to both a time-slot and a classroom. The solution for the Stage I model is given as input to the Stage II model, where certain irreversible decisions are fixed in Stage I. Tassopoulos and Beligiannis [72,73] developed and implemented swarm optimization based methods to solve the timetabling problem of high schools in Greece. Also, Valouxis