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Iterated local search based on multi-type perturbation for single-machine earliness/tardiness scheduling

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ABSTRACT

We propose an iterated local search based on a multi-type perturbation (ILS-MP) approach for single-machine scheduling to minimize the sum of linear earliness and quadratic tardiness penalties. The multi-type perturbation mechanism in ILS-MP probabilistically combines three types of perturbation strategies, namely tabu-based perturbation, construction-based perturbation, and random perturbation. Despite its simplicity, experimental results on a wide set of commonly used benchmark instances show that ILS-MP performs favourably in comparison with the current best approaches in the literature.

1. Introduction

Due to its theoretical challenge and practical relevance, single-machine earliness/tardiness scheduling has attracted considerable research attention of the scheduling community. In particular, it is related to the popular just-in-time (JIT) production philosophy and the practice of supply chain management. JIT production advocates that goods are produced only when they are needed, so both early and tardy deliveries are undesirable, which incur earliness and tardiness penalties, respectively. Thus, the ideal production schedule is one under which all the jobs are completed exactly on their due dates. Earliness/tardiness scheduling is also compatible with the practice of supply chain management, which emphasizes the coordination of material flows from suppliers to customers with a view to improving overall supply chain performance and providing better customer service. In general, the results and insights gained from single-machine scheduling research are applicable to more complex production environments such as the flow shop and job shop settings.

In this paper we consider the single-machine scheduling problem to minimize the sum of linear earliness and quadratic tardiness penalties (SMSP-LEQT), which is formally defined as follows: a set $\{J_1, J_2, \ldots J_n\}$ of independent jobs has to be scheduled on a single machine, which is continuously available from time zero onwards without idle time. The machine can process at most one job at a time and no preemption is allowed. Each job $J_j, j = 1, 2, \ldots, n$ has a processing time p_j and should be ideally completed on its due date d_j. The earliness and tardiness of job J_j are defined as $E_j = \max\{0, d_j - C_j\}$ and $T_j = \max\{0, C_j - d_j\}$, respectively, where C_j is the completion time of J_j. For SMSP-LEQT, we wish to find a schedule that minimizes $\sum_{j=1}^{n}(E_j + T_j^2)$ [15].

Similar single-machine scheduling problems with related objective functions have been studied in the literature. These include minimization of the sum of linear earliness and tardiness penalties, i.e., $\sum_{j=1}^{n}(E_j + T_j)$ [4,11,16], and minimization of quadratic lateness, i.e., $\sum_{j=1}^{n} L_j^2$ [8,12,14,17,18], where the lateness L_j of J_j is defined as $L_j = C_j - d_j$.

A generalization of the NP-hard single-machine scheduling problem to minimize the weighted tardiness [9], SMSP-LEQT is NP-hard. Both heuristic and exact methods have been applied to tackle this problem. Valente [20] presents a lower bounding procedure based on relaxation of the jobs' completion times and a branch-and-bound algorithm that adopts the proposed lower bound, together with an insertion-based dominance test. Schaller [16] introduces several branch-and-bound procedures for the problem, which are developed by incorporating a tighter lower bound and examining two dominance conditions. Among the heuristic approaches are the dispatching heuristics by Valente [19], which consider linear early/quadratic tardy dispatching rules, as well as a greedy-type procedure. Valente [21] proposes several heuristic algorithms based on the beam search technique. These include the classical beam search procedure, with both priority and total cost evaluation functions, as well as the filtered and recovering variants. Valente and Gonçalves [22] suggest several genetic algorithms based on a random key alphabet, which differ in the generation of the initial population and in the use of local search. Valente and Schaller [23] present some other heuristics for

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the considered problem, for both versions with and without idle time.

In this paper we propose a heuristic algorithm for SMSP-LEQT that follows the general framework of Iterated Local Search (ILS) [10] and adopts multi-type perturbation from Breakout Local Search (BLS) [1–3]. In essence, the basic idea of our ILS approach based on multi-type perturbation (ILS-MP) is to use local search to find local optima and apply a multi-type perturbation mechanism that probabilistically selects among three types of perturbation strategies that introduce different degrees of diversification to the search space. While one of the perturbation types is based on random moves, the other two are based on the principles that underpin the Tabu Search (TS) [5–7] and Greedy Randomized Adaptive Search Procedure (also known as GRASP [13]) metaheuristics.

We summarize the merits of the proposed ILS-MP algorithm as follows:

- Compared with the state-of-the-art algorithms in the literature for solving SMSP-LEQT, such as MA_IN, ILS-MP is conceptually simple and easy to implement. Indeed, while the reference algorithms often apply hybrid or evolutionary methods, ILS-MP is based on a simple local search working in conjunction with three dedicated perturbation operators.
- ILS-MP achieves better performance compared with the reference algorithms in the literature in terms of both solution quality and computational efficiency in many cases.

The rest of the paper is organized as follows: in Section 2 we describe in detail the proposed ILS-MP for SMSP-LEQT. We provide extensive computational evaluations and comparisons of ILS-MP with the current state-of-art approaches for SMSP-LEQT in Section 3. Finally, we conclude the paper in Section 4.

2. Iterated local search based on multi-type perturbation

2.1. General procedure

Following the general framework of ILS, our ILS-MP alternates iteratively between a local search phase (to reach local optima) and a dedicated perturbation phase (to discover new promising search spaces). Specifically, starting with an initial solution S_0, ILS-MP applies the steepest descent procedure (see Section 2.3) to reach a local optimum S. As explained in Section 2.3, each iteration of the steepest descent performs an exhaustive search in one of the randomly selected neighbourhoods (an insert neighbourhood or a swap neighbourhood), and selects the best-improving neighbouring solution. If such a solution does not exist in the current neighbourhood, local optimality is reached. At this point, ILS-MP triggers its multi-type perturbation mechanism, which first probabilistically selects among three different types of perturbation moves (tabu-based, construction-based, or random), and then applies a given number of moves of the selected type to the current local optimum S. This perturbed solution becomes a new starting point for the next phase of the descent. The best overall solution S_{best} is kept as the result. The proposed algorithm terminates once the number of consecutive local search phases with no improvement of S_{best} reaches a certain threshold *stop_iter*.

The general framework of the ILS-MP algorithm is outlined in Algorithm 1. The following sections detail the main algorithmic components of ILS-MP.

Algorithm 1. ILS-MP for SMSP-LEQT.

```
Generate an initial solution $S'$ /* See Section 2.2 */
$S_{best} = S'$
$iter\_no\_improv \leftarrow 0$
while $iter\_no\_improv < stop\_iter$ do
    $S \leftarrow LocalSearch(S')$ /* See Section 2.3 */
    if $S$ is better than $S_{best}$ then
        $S_{best} \leftarrow S$
        $iter\_no\_improv \leftarrow 0$
    else
        $iter\_no\_improv \leftarrow iter\_no\_improv + 1$
    end if
    if $(rand(0, 0.1, \dots, 1) < P)$ /*$P \in [0, 0.01, \dots, 1]$ is a coeff.*/ then
        $S' = TabuBasedPerturbation(S, L_1)$ /* See Section 2.5 */
    else if $(rand(0, 0.01, \dots, 1) < (1-P) \cdot Q)$ /*$Q \in [0, 0.01, \dots, 1]$ is a coeff.*/ then
        $S' = ConstructionBasedPerturbation(S, L_2)$ /* See Section 2.5 */
    else
        $S' = RandomPerturbation(S, L_3)$ /* See Section 2.5 */
    end if
end while
return $S_{best}$
```

2.2. Initial solution

Given an unordered set of n independent jobs $S = \{J_1, J_2, \dots, J_n\}$, ILS-MP uses a greedy heuristic procedure to generate a starting point for the search, i.e., an ordered set S' of n jobs. Initially, $S' = \varnothing$. Starting from scratch, we randomly select a job J_i from S, let $S = S - \{J_i\}$, and insert J_i into the best position in S' according to the evaluation strategy (see Section 2.4). Then, $S' = S' \cup \{J_i\}$. The above procedure is repeated until $S = \varnothing$.

Note that we carried out additional experiments by using the NEH heuristic to generate the initial solution. However, the results show that there is no significant difference in terms of the best results found. The reason might lie in the fact that ILS-MP makes use of several diversification mechanisms to diversify the search when it gets stuck in a local optimum trap. So the performance of ILS-MP does not rely much on the quality of the initial solution. This is also one of the advantages of ILS-MP.

2.3. Neighbourhood relations and their exploitation

To move from one solution to another in the search space, the local search procedure of ILS-MP employs two different neighbourhood relations:

Insert: A job $J_i \in S$ $(1 \le i \le n)$ is removed from its current position in the schedule S and inserted immediately before (forwards) or after (backwards) another job J_k $(k \ne i)$, thus producing a total of $n-1$ possible positions (i.e., solutions). Examples of insertion forwards and backwards are provided in Figs. 1 and 2, respectively.

Swap: Two jobs swap their positions, which generates a total of $n(n-1)/2$ possible solutions. An example of a swap move is given in Fig. 3.

To reduce the size of the neighbourhood, the local search in ILS-MP algorithm applies distance restrictions for both insert and swap moves. The distance for an insert move is the absolute

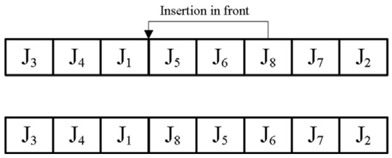

Fig. 1. Insertion forwards. J_8 is inserted immediately before J_5.