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Multi-period Vehicle Routing Problem with Due dates

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ABSTRACT

In this paper we study the Multi-period Vehicle Routing Problem with Due dates (MVRPD), where customers have to be served between a release and a due date. Customers with due dates exceeding the planning period may be postponed at a cost. A fleet of capacitated vehicles is available to perform the distribution in each day of the planning period. The objective of the problem is to find vehicle routes for each day such that the overall cost of the distribution, including transportation costs, inventory costs and penalty costs for postponed service, is minimized. We present alternative formulations for the MVRPD and enhance the formulations with valid inequalities. The formulations are solved with a branch-and-cut algorithm and computationally compared. Furthermore, we present a computational analysis aimed at highlighting managerial insights. We study the potential benefit that can be achieved by incorporating flexibility in the due dates and the number of vehicles. Finally, we highlight the effect of reducing vehicle capacity.

1. Introduction

In Vehicle Routing Problems (VRPs) the transportation planning period is a day and the service day of customers is assumed to be known. In a given day customers have to be assigned to vehicles and the order of visits of each vehicle has to be determined.

Several situations exist where some flexibility on the service time is possible but the quantities to be delivered are fixed. This is the case when customers make orders and the delivery is guaranteed within a certain number of days. This is in fact one of the most common situations. Often contracts are established between supplier and customers whose cost depends on the time-to-delivery. The shorter the time-to-delivery the more expensive the contract is. Similarly, in e-commerce, customers make orders and a due date is established at the time an order is made. The time of service is a decision variable while the quantities to be delivered are given.

The real problem that motivated this study arises in city logistics. City logistics aims to reduce the nuisances associated to freight transportation in urban areas. A study on ad-hoc freight transportation systems for congested urban areas was presented in [13] while in [14] different models are presented for the evaluation and planning of city logistic systems. For a recent reference on a heuristic algorithm for a vehicle routing problem arising in city logistics we refer to [20].

There are different settings in city logistic systems. The one we consider in this paper is composed by a central distribution center (CDC) which is used to consolidate distribution activities within an urban environment. Customers are private citizens, offices or shops. Customers have made orders and request the delivery to take place within a given due date. Trucks deliver goods to the CDC where they are consolidated in vehicles dedicated for conducting urban distribution activities. The problem is how to organize the distribution of goods to final customers. Goods have to be distributed from the CDC to the customers within the due dates in such a way that the distribution cost is minimized. We refer to this problem as the Multi-period Vehicle Routing Problem with Due dates (MVRPD), where a period corresponds to a day.

The MVRPD conceptually lies between the Periodic Vehicle Routing Problem (PVRP) and the Inventory Routing Problem (IRP). In the PVRP the planning period is made of a certain number of days. A customer may request to be served one or more times in the planning period. Alternative sequences of days of visit are predefined for each customer. Given a sequence of days of visit, the quantities to be delivered in each day of visit are known. For example the planning period may be made of 6 days. A customer may require two visits in a week and its possible alternative sequences may be (1,4), (2,5), (3,6). The problem becomes that of choosing for each customer one sequence of days and, for each day, assigning customers to vehicles and determining for each vehicle the order of visit. Therefore, the PVRPs model the situation of customers requesting a certain frequency of service with the flexibility of choosing the precise days of service. The customers determine the service frequency and the quantities to be delivered. The flexibility in the choice

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of the precise sequence of days of service creates saving opportunities, but makes the problem harder to be solved. A commonly used formulation is provided in [9]. A comprehensive survey on the PVRP and its extensions can be found in [19]. The exact algorithm proposed in [4] is currently the leading methodology for the exact solution of the PVRP. For a recent reference on a heuristic algorithm for the PVRP we refer to [22]. The issue of allowing more flexibility in PVRP is studied in [17] where the PVRP with Service Choice (PVRP-SC) is introduced, that is a PVRP in which service frequency is a decision of the model. The authors propose a mathematical formulation and an exact solution approach for the problem in [17] while in [16] a continuous approximation model for the same problem is proposed. In [18] the authors developed a tabu search method for the PVRP that can incorporate a range of operational flexibility options, like the possibility to increase the set of visit schedules, decide visit frequency, vary the driver who visits a customer and decide delivery amounts per visit. The authors analyze the trade-offs between the system performance improvements due to operational flexibility and the implementation, computational and modeling complexity. They introduce quantitative measures in order to evaluate the complexity increase and provide insights both from a managerial and a modeling perspective.

In the IRPs, the planning period is made of a certain number of days, as in the PVRPs. However, the IRP includes more flexibility with respect to the PVRP. A customer may be visited any number of times and the quantities to be delivered have to be determined. The customer consumption is known, day by day, but, contrary to the VRPs and the PVRPs, the quantities to be delivered are not. The days of service and the quantities to be delivered are to be determined in such a way that a stock-out never occurs at any customer. In addition, as in VRPs and in PVRPs, for each day customers have to be assigned to vehicles and the order of visit has to be determined. Additional savings can be achieved with respect to VRPs and also with respect to PVRPs. IRPs are interesting and challenging problems even when there is only one destination, i.e., when the routing side of the problem is trivially solved. An introduction to IRPs with a focus on the case of one origin and one destination can be found in [7], while a tutorial for the case of multiple destinations has been published shortly after in [8]. Surveys are also available, the most recent ones being [6,10].

The IRPs model different practical situations where the decision space is very broad. In particular, they model a management practice which is known as Vendor Managed Inventory (VMI). In VMI the supplier has regular information on the status of the inventory levels of its customers and of their consumptions and has the freedom to organize the distribution, provided that it guarantees no stock-out occurs at the customers. In the most basic IRP, customers are to be supplied over a certain number of days by a fleet of capacitated vehicles, based on a depot. Their consumption is known, day by day. Each customer has a maximum inventory capacity. Different replenishment policies may be adopted. The quantity delivered to a customer may be such that the inventory capacity is reached (Order-Up to level policy) or such that the inventory capacity is not exceeded (Maximum Level policy). The decisions include when to serve each customer (how many times and the precise days), how much to deliver when a customer is served and the routes followed by the vehicles. This problem was introduced in [5]. The first exact method for the solution of this problem was proposed in [2] for the case of one vehicle. Exact algorithms for the multi-vehicle extension were recently presented in [11,12,15], while alternative formulations are compared in [3].

The decision space of the MVRPD is broader than that of the VRP, as the days of service have to be chosen, and more restricted than in IRPs, as the quantities are given. The MVRPD are close to the PVRPs but typically there is no periodicity in the service. Furthermore, we mention that in [1,24] the authors study the dynamic multi-period vehicle routing problem, where customers' requests arrive dynamically over time and must be satisfied within a time window. The latter comprises several time periods of the planning horizon and thus resembles the due date in the MVRPD. In [24] the objective function comprises travelling cost, waiting time and balancing daily workload. In [1] the objective is to minimize travelling cost in a stochastic setting.

The contributions of this paper are fourfold. We first introduce the MVRPD. We investigate three alternative formulations and propose a set of valid inequalities for each one that exploit the problem structure. Each formulation is solved with a branch-and-cut algorithm and we identify the best one through computational experiments. Finally, we evaluate the impact of altering due dates, number of vehicles and vehicle capacity. Our analysis provide valuable managerial insights.

The rest of the paper is organized as follows. In Section 2 we develop three formulations together with valid inequalities. In Section 3 we present our computational experiments. Finally, we provide concluding remarks in Section 4.

2. Problem description and formulations

We consider a planning horizon, composed of a certain number of days. A set of customers have to be served. Each customer has placed an order that has to be satisfied within a certain due date. Multiple orders of the same customers may be modelled through different co-located customers. In the following, we will use the terms 'order' and 'customer' with the same meaning. A fleet of capacitated vehicles, based on a depot, are available to serve the customers. The goods requested by a customer may not be available at the beginning of the planning horizon but are known to become available at a later time. If the due date of a customer exceeds the planning horizon, its service may be postponed. In this case, a penalty will be charged. The latter cost is assumed to encompass the inventory holding cost of customers beyond the planning horizon. The problem is to design daily distribution routes for the given planning horizon. We refer to this problem as the Multi-period Vehicle Routing Problem with Due dates (MVRPD).

A planning horizon $T=\{1,\dots,H\}$ is given. The MVRPD is defined on a complete directed graph $G=(V,A)$, where $V=\{1,\dots,n\}$ is the vertex set and $A=\{(i,j): i,j\in V\}$ is the arc set. Vertex 1 is the depot at which m identical vehicles of capacity Q are based, whereas the remaining vertices represent customers. An order quantity q_i is associated with customer i, together with a release date r_i, $1\le r_i\le H$ and a due date d_i, $d_i\ge r_i$. The due date may exceed the planning horizon. If it does, the customer may be served within the planning horizon or its service may be postponed. A penalty cost p_i is charged if customer i is postponed. A nonnegative cost c_{ij} is associated with each arc $(i,j)\in V$ and represents the transportation cost incurred by travelling directly from i to j. For all periods $t\in T$ routes are constructed such that each customer order is delivered at most once by one vehicle (exactly once if the due date does not exceed the planning horizon), all routes start and end at the depot and the total quantity on any route does not exceed the vehicle capacity Q. Furthermore, the routes must be such that each customer is not served before its release date. For each customer i, an inventory holding cost h_i is charged for each day that order i spends at the depot. We assume that the depot has sufficient capacity to hold the entire demand delivered to it. We call the period $[r_i,d_i]$ the window associated with customer i. Let $C\subset\{V\setminus\{1\}\}$ be the set of customers whose due date is greater than H. Each order $i\in C$, if not served within H, incurs a holding cost $h_i[H-r_i]$ as well as the penalty cost p_i.