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Dynamical analysis of Cohen-Grossberg neural networks with time-delays and impulses☆

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ABSTRACT

A model describing the dynamics of Cohen-Grossberg neural networks with time-delays and impulses is considered. By means of Lyapunov functionals and a differential inequality technique, criteria on global exponential stability of this model are derived. Many adjustable parameters are introduced in the criteria to provide flexibility for the design and analysis of the system. The results of this paper are new and they supplement previously known results.

1. Introduction

The theory of impulsive delay differential equations is emerging as an important area of investigation, since it is a lot richer than the corresponding theory of nonimpulsive delay differential equations. Many evolution processes in nature are characterized by the fact that at certain moments of time they experience an abrupt change of state. That was the reason for the development of the theory of impulsive differential equations and impulsive delay differential equations, see the monographs [1,2].

The purpose of this paper is to study the stability of the following impulsive Cohen–Grossberg neural networks (CGNNS) with variable coefficients and several time-varying delays:

$$\begin{cases} \dot{x}_i(t) = -a_i(x_i(t))\Big[b_i(t, x_i(t)) - \sum_{j=1}^{n} c_{ij}(t)f_j(x_j(t)) - \sum_{j=1}^{n} d_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) + J_i(t)\Big], & \\ \quad \text{a.e. } t \geq 0, t \neq t_k, & \text{(a)} \\ x_i(t^+) = g_{ik}(x_i(t)) + h_{ik}(x_i(t-\varsigma_i(t))) + I_{ik}(t), \quad t = t_k, i = 1, 2, \ldots, n; k = 1, 2, \ldots, & \text{(b)} \end{cases} \tag{1.1}$$

where n corresponds to the number of units in a neural network; for $i, j = 1, 2, \ldots, n$, $x_i(t)$ denotes the potential of cell i at time t; $0 \leq \tau_{ij}(t), \varsigma_i(t) \leq \tau$ correspond to the transmission delays. The first part (called the continuous part) of (1.1) describes the continuous evolution processes of the neural networks. For $i, j = 1, 2, \ldots, n$, a_i represents an amplification function; b_i is an appropriately behaved function; $c_{ij}(t)$ and $d_{ij}(t)$ denote the strengths of connectivity between cell i and j at time t, respectively; f_i shows how the ith neuron reacts to the input; $J_i(t)$ is the external bias on the ith neuron at time t. The second part (called the discrete part) of (1.1) describes that the evolution processes experience an abrupt change of

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states at the moments of t_k (called impulsive moments); For $i = 1, 2, \ldots, n$; $k = 1, 2, \ldots$, the fixed moments of t_k, satisfy $t_1 < t_2 < \cdots < t_n < \cdots$, and $\lim_{k\to\infty} t_k = \infty$; g_{ik} represents impulsive perturbations of ith unit at time t_k; h_{ik} represents impulsive perturbations of ith unit at time t_k, which is caused by the transmission delays; $I_{ik}(t_k)$ represents the external impulsive input at time t_k.

(1.1)(a) as a model of neural network (CGNNS), which included Hopfield neural networks as a special case, has been studied widely. Recently for the delayed CGNNS such as (1.1)(a) with $b_i(t, x_i(t)) = b_i(x_i(t))(i = 1, 2, \ldots, n)$, some criteria for the global asymptotic stability are established. We refer to [3–6]. In [7], the authors investigate the effects of delays, but the differentiability of the varying delays τ_{ij} and the behaved function b_i are needed. However their work mostly focuses on the autonomous CGNNS. In [8,9], the authors study the existence and exponential stability of periodic solutions for a periodic nonautonomous CGNNS, respectively. Other results for a nonautonomous CGNNS are few. We refer to [10–12].

In this paper, we will investigate the global asymptotic stability of the nonautonomous CGNNS and focus on the effect of impulses on the dynamic behavior of (1.1). The results of this paper are new and they supplement previously known results.

For a continuous function $a(t)$ defined on $R_+ = [0, \infty)$, we denote $a^+(t) = \max_{t\in R_+}\{0, a(t)\}$ and $a^-(t) = \min_{t\in R_+}\{0, a(t)\}$.

For convenience, the following conditions are listed.

(H_1) a_i and f_i $(i = 1, 2, \ldots, n)$ are continuous on R; c_{ij} and J_i $(i, j = 1, 2, \ldots, n)$ are continuous on R_+; d_{ij} $(i, j = 1, 2, \ldots, n)$ is continuous and bounded on R_+; $b_i(i = 1, 2, \ldots, n)$ is continuous on $R_+ \times R$; Furthermore, there exist positive constants $\underline{\alpha}_i$ and $\bar{\alpha}_i$ such that $\underline{\alpha}_i \le a_i(x) \le \bar{\alpha}_i$ for all $x \in R$ and $i = 1, 2, \ldots, n$.

(H_2) There exist positive continuous functions $\beta_i(t)$, $i = 1, 2, \ldots, n$, such that

$$\frac{b_i(t, u) - b_i(t, v)}{u - v} \ge \beta_i(t) > 0 \quad \text{for all } t \in [0, \infty), u, v \in R \text{ and } u \ne v;$$

(H_2^*) There exist positive continuous functions $\beta_i(t)$, $i = 1, 2, \ldots, n$, such that

$$ub_i(t, u) \ge \beta_i(t)u^2 \quad \text{for all } t \in [0, \infty), u \in R;$$

(H_3) There are positive constants $F_i > 0$, $i = 1, 2, \ldots, n$, such that

$$|f_i(u) - f_i(v)| \le F_i|u - v|,$$

for all $u, v \in R$ and $i = 1, 2, \ldots, n$.

(H_4) There exist positive constants $q_k > 0$, $p_i > 0$, $\mu_{ki} \in R$, $\omega_{ki} \in R(i = 1, 2, \ldots, n; k = 1, 2, \ldots, m)$, $r > 1$ and $\sigma > 0$ such that $\sum_{k=1}^m q_k = r - 1$ and

$$r\underline{\alpha}_i\beta_i(t) - \sum_{j=1}^n\sum_{k=1}^m \bar{\alpha}_j q_k\left(c_{ij}^+(t)F_j^{\frac{r\mu_{kj}}{q_k}} + d_{ij}^+(t)F_j^{\frac{r\omega_{kj}}{q_k}}\right) - \frac{1}{p_i}\sum_{j=1}^n \bar{\alpha}_j p_j\left(c_{ij}^+(t)F_j^{r(1-\sum_{k=1}^m \mu_{kj})} + d_{ij}^+(t)F_j^{r(1-\sum_{k=1}^m \omega_{kj})}\right) \ge \sigma > 0,$$

for $t \in [0, \infty)$ and $i = 1, 2, \ldots, n$.

(H_4^*) There exist positive constants $p_1, p_2, \ldots, p_n$ and σ such that

$$\underline{\alpha}_i\beta_i(t) - \frac{1}{p_i}\sum_{j=1}^n \bar{\alpha}_j p_j F_j\left(c_{ij}^+(t) + d_{ij}^+(t)\right) \ge \sigma > 0, \quad \text{for all } t \in [0, \infty) \text{ and } i = 1, 2, \ldots, n.$$

(H_5) There exist positive constants G_{ik} and H_{ik} such that

$$|g_{ik}(u) - g_{ik}(v)| \le G_{ik}|u - v|, |h_{ik}(u) - h_{ik}(v)| \le H_{ik}|u - v|, \max_{1\le i\le n, 1\le k} \frac{\bar{\alpha}_i}{\underline{\alpha}_i}H_{ik} + \max_{1\le i\le n, 1\le k} \frac{\bar{\alpha}_i}{\underline{\alpha}_i}G_{ik} < 1$$

for all $u, v \in R$ and $i = 1, 2, \ldots, n$; $k = 1, 2, \ldots$.

Define

$$PC([-\tau, 0], R) = \Big\{\hat{\psi} : [-\tau, 0] \to R|\hat{\psi}(t^-) = \hat{\psi}(t), \text{ for } t \in [-\tau, 0], \hat{\psi}(t^+) \text{ exists and } \hat{\psi}(t^+) = \hat{\psi}(t) \text{ for all but at most a finite number of points } t \in [-\tau, 0].\Big\},$$

$$PC([-\tau, 0], R^n) = \Big\{\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T|\psi_i \in PC([-\tau, 0], R), i = 1, 2, \ldots, n.\Big\}.$$

For any $\hat{\psi} \in PC([-\tau, 0], R)$, $\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T \in PC([-\tau, 0], R^n)$, define $\|\cdot\|_\tau$ and $\|\cdot\|_\tau^n$ as $\|\hat{\psi}\|_\tau = \sup_{-\tau\le s\le 0}|\hat{\psi}(s)|$ and $\|\psi\|_\tau^n = \max_{1\le i\le n}\|\psi_i\|_\tau$, respectively.

Moreover, we define $x_t \in PC([-\tau, 0], R^n)$ by $x_t(s) = x(t + s)$ for $-\tau \le s \le 0$.

We assume that (1.1) has the following initial conditions

$$x_i(s) = \phi_i(s), \quad \text{for } -\tau \le s \le 0, \tag{1.2}$$

where $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^T \in PC([-\tau, 0], R^n)$. According to [13], the initial value problem (1.1) and (1.2) has the unique solution $x(t, \phi)$ under assumptions (H_3) and (H_5).