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Sufficiency and duality for multiobjective variational control problems with G-invexity

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ABSTRACT

Antczak introduced vector-valued G-invex functions in 2009, which is a new class of generalized convex functions for differentiable multiobjective programming problems. In this paper, we extend the vector-valued G-invex functions to multiobjective variational control problems. By using the new concepts, a number of sufficient optimality results and Mond–Weir type duality results are obtained for multiobjective variational control programming problems.

1. Introduction

Optimal control problems are important problems, which have been widely used in industrial process control, flight control design and other diverse fields. From 1964, a number of duality results for single objective control problem have been obtained in the literature; see [1–3] and the references therein. In 1988, Mond and Smart [4] obtained duality results for control problems under invex conditions. Bhatia and Kumar [5] extended the work of Mond and Hanson [3] to multiobjective control problems and obtained duality results under ρ-invexity assumptions and its generalizations. Under invexity conditions, Chen [6] studied duality theorems for multiobjective control problems.

In recent years, Nahak and Nanda [7] studied duality results for multiobjective variational control problems under generalized (F, ρ)-convexity conditions. Under the same conditions as [7], Patel [8] formulated Wolfe and Mond–Weir type duals for multiobjective variational control problems and obtained weak and strong duality theorems by using the concept of efficiency. Nahak and Nanda [9] established sufficiency optimality criteria and duality results for multiobjective variational control problems under V-invexity assumptions. Recently, Ahmad and Sharma [10] extended the concept of $(F, \alpha, \rho, \theta)$-$V$-convexity to variational control problems and obtained sufficient optimality conditions, Wolfe type and Mond–Weir type duality results for multiobjective variational control problems. Patel [11] extended the class of V-univex type I functions and their generalizations to multiobjective variational control problems and established sufficiency and mixed type duality results under generalized V-univexity type I conditions.

In very recent years, Antczak [12] introduced a new class of vector-valued G-invex functions and their generalizations. Antczak also established some sufficiency conditions and several duality [13] results for multiobjective programming under vector-valued G-invexity requirements.

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In this paper, we extend the concept of vector-valued G-invexity in [12,13] to multiobjective variational control problems. We establish various sufficiency optimality criteria and duality results for multiobjective variational control programming problems under the assumptions of vector-valued G-invexity and their generalizations.

2. Preliminaries

Let $\mathbb{R}^n$ be n-dimensional Euclidean space, and $\mathbb{R}^n_+$ be its nonnegative orthant. For any $x = (x_1, x_2, \ldots, x_n)^T$, $y = (y_1, y_2, \ldots, y_n)^T$ be in $\mathbb{R}^n$. We define

(i) $x = y$ if and only if $x_i = y_i$ for all $i = 1, 2, \ldots, n$;
(ii) $x < y$ if and only if $x_i < y_i$ for all $i = 1, 2, \ldots, n$;
(iii) $x \leqq y$ if and only if $x_i \leq y_i$ for all $i = 1, 2, \ldots, n$;
(iv) $x \leq y$ if and only if $x \leqq y$ and $x \neq y$.

Let $I = [a, b]$ be a real interval and $f : I \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^m \longrightarrow \mathbb{R}^p$, $g : I \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^m \longrightarrow \mathbb{R}^m$ and $h : I \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^m \longrightarrow \mathbb{R}^n$ be continuously differentiable functions. We denote $M = \{1, 2, \ldots, m\}$ and $K = \{1, 2, \ldots, n\}$. Consider the function $f(t, x(t), \dot{x}(t), u(t), \dot{u}(t))$, where $x : I \to \mathbb{R}^n$ with derivative $\dot{x}$ and $u : I \to \mathbb{R}^m$ with derivative $\dot{u}$, t is the independent variable, x is the state variable and u is the control variable. $u(t)$ is related to $x(t)$ through the state equation $h(t, x(t), \dot{x}(t), u(t), \dot{u}(t)) = 0$. The symbol $()^T$ denotes for the transpose. For a real function $f(t, x(t), \dot{x}(t), u(t), \dot{u}(t))$, $f^i_t, f^i_x, f^i_{\dot{x}}, f^i_u$ and $f^i_{\dot{u}}$ denote the partial derivative of f_i, $i \in P = \{1, 2, \ldots, p\}$ with respect to t, x, $\dot{x}$, u and $\dot{u}$, respectively. For example,

$$f^i_x = \left(\frac{\partial f_i}{\partial x_1}, \frac{\partial f_i}{\partial x_2}, \ldots, \frac{\partial f_i}{\partial x_n}\right), \qquad f^i_{\dot{x}} = \left(\frac{\partial f_i}{\partial \dot{x}_1}, \frac{\partial f_i}{\partial \dot{x}_2}, \ldots, \frac{\partial f_i}{\partial \dot{x}_n}\right)$$
$$f^i_u = \left(\frac{\partial f_i}{\partial u_1}, \frac{\partial f_i}{\partial u_2}, \ldots, \frac{\partial f_i}{\partial u_m}\right), \qquad f^i_{\dot{u}} = \left(\frac{\partial f_i}{\partial \dot{u}_1}, \frac{\partial f_i}{\partial \dot{u}_2}, \ldots, \frac{\partial f_i}{\partial \dot{u}_m}\right).$$

Similarly, $g^j_t, g^j_x, g^j_{\dot{x}}, g^j_u, g^j_{\dot{u}}$ and $h^k_t, h^k_x, h^k_{\dot{x}}, h^k_u, h^k_{\dot{u}}$ can be defined.

Let $S(I, \mathbb{R}^n)$ denote the space of piecewise smooth functions x with norm $\|x\| = \|x\|_\infty + \|Dx\|_\infty$, where the differentiation operator D is given by

$$u = Dx \Longleftrightarrow x(t) = \alpha + \int_a^t u(s)ds,$$

where α is a given boundary value. Therefore, $D = \frac{d}{dt}$ except at discontinuities. Let X, Y denote the space of all piecewise smooth functions $x : I \to \mathbb{R}^n$ and $u : I \to \mathbb{R}^m$. Let $F^i : X \times Y \longrightarrow \mathbb{R}$ defined by $F_i(x(t), u(t)) = \int_a^b f^i(t, x(t), \dot{x}(t), u, \dot{u}(t))dt$ be Frechet differentiable. For notational convenience, we use $f(t, x, \dot{x}, u, \dot{u})$ for $f(t, x(t), \dot{x}(t), u(t), \dot{u}(t))$.

In 2009, Antczak [12,13] introduced the concept of vector-valued G-invex function. Let $f = (f_1, f_2, \ldots, f_p) : C \to \mathbb{R}^p$ be a vector-valued differentiable function defined on a nonempty open set $C \subset \mathbb{R}^n$, and $I_{f_i}(C)$, $i = 1, \ldots, p$, be the range of f_i, that is, the image of C under f_i.

Definition 2.1 (*[12]*). Let $f : C \to \mathbb{R}^p$ be a vector-valued differentiable function defined on a nonempty set $C \subset \mathbb{R}^n$ and $u \in C$. If there exist a differentiable vector-valued function $G_f = (G_{f_1}, \ldots, G_{f_p}) : \mathbb{R} \to \mathbb{R}^p$ such that any its component $G_{f_i} : I_{f_i} \to \mathbb{R}$ is a strictly increasing function on its domain and a vector-valued function $\eta : C \times C \to \mathbb{R}^n$ such that, for all $x \in C (x \neq u)$ and for any $i = 1, \ldots, p$,

$$G_{f_i}(f_i(x)) - G_{f_i}(f_i(u)) - G'_{f_i}(f_i(u))\nabla f_i(u)\eta(x, u) \geq 0 \quad (>),$$

then f is said to be a (strictly) vector G_f-invex function at u on C with respect to η. If the inequality is satisfied for each $u \in C$, then f is vector G_f-invex on C with respect to η.

From now onward, we establish some classes of new G-invex functions.

Definition 2.2. A vector function $F = (F^1, F^2, \ldots, F^p)$ is said to be G-invex at $(x^*, u^*) \in (X, Y)$, if there exist differentiable functions $\eta(t, x, x^*, u, u^*) : I \times X \times X \times Y \times Y \to \mathbb{R}^n$ with $\eta(t, x, x^*, u, u^*) = 0$ at t if $x(t) = x^*(t)$, $\xi(t, x, x^*, u, u^*) : I \times X \times X \times Y \times Y \to \mathbb{R}^m$ and a differentiable vector-valued function $G_f = (G_{f_1}, \ldots, G_{f_p}) : \mathbb{R} \to \mathbb{R}^p$ such that any its component $G_{f_i} : I_{f_i}(C) \to \mathbb{R}$ is a strictly increasing function on its domain, for each $x, x^* \in X$, $u, u^* \in Y$, and for $i = 1, 2, \ldots, p$

$$G_{f_i}(F_i(x, u)) - G_{f_i}(F_i(x^*, u^*)) \geq G'_{f_i}(F_i(x^*, u^*)) \int_a^b \Big\{ \eta(t, x, x^*, u, u^*)^T (f^i_x(t, x^*, \dot{x}^*, u^*, \dot{u}^*) - \frac{d}{dt} f^i_{\dot{x}}(t, x^*, \dot{x}^*, u^*, \dot{u}^*)) + \xi(t, x, x^*, u, u^*)^T (f^i_u(t, x^*, \dot{x}^*, u^*, \dot{u}^*) - \frac{d}{dt} f^i_{\dot{u}}(t, x^*, \dot{x}^*, u^*, \dot{u}^*)) \Big\} dt.$$