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Impulsively control complex networks with different dynamical nodes to its trivial equilibrium[☆]

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ABSTRACT

This paper investigates the stability of complex networks with different dynamical nodes by impulsive control. A model of complex network with different dynamical nodes is presented and then the impulsive controlled network is written as a whole vector equation. Specially, the coupling matrix in this model is not assumed to be diffusive or irreducible. Some criteria and corollary are derived for the presented impulsive controlled complex networks. Furthermore, the results are illustrated by a complex network composed of the chaotic Lorenz systems and Chua's circuit systems. All the numerical simulations verify the correctness of the theoretical results.

1. Introduction

Recently, the control and synchronization of complex networks has been a focus for many scientists from various fields, for instance, sociology, biology, mathematics and physics [1–6]. And several different approaches including adaptive synchronization [7], robust synchronization [8] and impulsive control [9] have been introduced to solve the above problem. Among these approaches, it has been proved that the impulsive control method is effective and relatively easily realized [10,11]. It allows stability of a complex network only by small impulses being sent to the receiving systems at the discrete impulsive instances, which can reduce the information redundancy in the transmitted signal and increase robustness against the disturbances. In this sense, impulsive control schemes have been applied to numerous chaos-based communication systems for cryptographically secure purposes and detailed experiments have been carried out [12–14].

Over the past decades, a great number of natural complex networks— such as cooperate networks, social networks, neural networks, WWW, food webs, electrical power grids and so on have been widely studied by researchers. However, most of the network models consist of the same dynamical nodes and the coupling matrices are often assumed to be diffusive and irreducible in the existing literatures [3,6,15]. Little work has been done for the networks of different dynamical nodes with general coupling matrices.

In this letter, we investigate the issue on the stability of complex networks with different dynamical nodes by impulsive control. Firstly, a model of complex networks with different dynamical nodes is presented, in which the coupling matrix is not assumed to be diffusive or irreducible. Then, the impulsive controlled network is written as a whole vector equation by introducing the Kronecker product. Some criteria and corollary are obtained for the presented impulsive controlled complex networks. Finally, the results are illustrated by a complex network composed of the chaotic Lorenz systems and Chua's circuit systems. All involved numerical simulations verify the effectiveness of the theoretical analysis.

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2. A model of complex networks with different dynamical nodes

In the following study, we consider a complex network consisting of different kinds of dynamical nodes. For the convenience of clarification, we assume there are two kinds of dynamical nodes in this network. Each node of the networks is an n-dimensional autonomous dynamical system. The state equations of the entire networks is described by

$$\begin{cases} \dot{x}_i = f(x_i) + \sum_{j=1}^{N} c_{ij} A x_j, & i = 1, 2, \ldots, l, \\ \dot{x}_i = g(x_i) + \sum_{j=1}^{N} c_{ij} A x_j, & i = l+1, l+2, \ldots, N, \end{cases} \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^{\mathrm{T}} \in \mathbf{R}^n$ is a state vector representing the state variables of node i, and $f, g : \mathbf{R}^n \to \mathbf{R}^n$ are continuous nonlinear vector valued functions and $f(0) = 0$, $g(0) = 0$. The matrix $C = (c_{ij})_{N\times N}$ is the coupling configuration matrix of the networks, $A \in \mathbf{R}^{n\times n}$ is the inner connecting matrix in each node.

For simplicity of further discussion, we separate the linear part from the nonlinear part of f, g as

$$f(x_i) = Fx_i + \phi(x_i), \quad i = 1, 2, \ldots, l, \tag{2}$$

$$g(x_i) = Gx_i + \psi(x_i), \quad i = l+1, l+2, \ldots, N, \tag{3}$$

where $F, G \in \mathbf{R}^{n\times n}$ are the corresponding constant matrices.

Then the network (1) can be rewritten by using the Kronecker product as

$$\dot{X} = DX + \Phi(X) + (C \otimes A)X \tag{4}$$

where

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_l \\ x_{l+1} \\ \vdots \\ x_N \end{pmatrix}, \quad \Phi(X) = \begin{pmatrix} \phi(x_1) \\ \phi(x_2) \\ \vdots \\ \phi(x_l) \\ \psi(x_{l+1}) \\ \vdots \\ \psi(x_N) \end{pmatrix}, \quad D = \begin{pmatrix} F & & & & & & \\ & F & & & & & \\ & & \ddots & & & & \\ & & & F & & & \\ & & & & G & & \\ & & & & & \ddots & \\ & & & & & & G \end{pmatrix}.$$

Now, the impulsive controlled network can be described as below

$$\begin{cases} \dot{X} = DX + \Phi(X) + (C \otimes A)X, & t \neq t_k, \\ \Delta X(t_k^+) = B_k X(t_k), & t = t_k, k = 1, 2, \ldots, \\ X(t_0^+) = X_0, \end{cases} \tag{5}$$

where the matrices $B_k \in \mathbf{R}^{nN\times nN}$ $(k = 1, 2, \ldots)$ are the impulsive feedback gain at the moment t_k. Moreover, $\Delta X(t_k^+) = X(t_k^+) - X(t_k^-)$, $X(t_k^+) = \lim_{t\to t_k^+} X(t)$ and any solution of (5) is left continuous at each t_k, i.e. $X(t_k^-) = X(t_k)$. The moments of impulse satisfy $t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ and $\lim_{k\to\infty} t_k = \infty$, $\tau_k = t_k - t_{k-1} < \infty$.

The objective of this paper is to obtain some sufficient conditions between the outer-coupling matrix C, the inner-coupling matrix A, the impulsive controller gain B_k, and impulse distances τ_k such that the origin of network (5) is stable.

3. Stability analysis of the presented dynamical networks

Throughout this paper, the following assumption will be required.

Assumption 1. There exists a constant $L > 0$ such that all the nonlinear functions $\phi(x_i)(i = 1, 2, \ldots, l)$, $\psi(x_i)(i = l+1, l+2, \ldots, N)$ satisfy

$$(\phi(x_i))^{\mathrm{T}} x_i \le L x_i^{\mathrm{T}} x_i, \quad i = 1, 2, \ldots, l, \tag{6}$$

and

$$(\psi(x_i))^{\mathrm{T}} x_i \le L x_i^{\mathrm{T}} x_i, \quad i = l+1, l+2, \ldots, N. \tag{7}$$

That is to say, $(\Phi(X))^{\mathrm{T}} X \le L X^{\mathrm{T}} X$.

In fact, there are many classical chaotic systems, such as Lorenz system, Chen system, Lü system and Chua's circuit system, their corresponding nonlinear functions all have the above quality.