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Hadamard product of certain meromorphic starlike and convex functions

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ABSTRACT

In this paper, the authors establish certain results concerning the Hadamard product for two classes related to starlike and convex univalent meromorphic functions of order α and type β with positive coefficients.

1. Introduction

Throughout this paper, let the functions of the form :

$$\varphi(z) = c_1 z - \sum_{n=2}^{\infty} c_n z^n \quad (c_1 > 0, c_n \geq 0), \tag{1.1}$$

and

$$\psi(z) = d_1 z - \sum_{n=2}^{\infty} d_n z^n \quad (d_1 > 0, d_n \geq 0) \tag{1.2}$$

be regular and univalent in the unit disc $U = \{z : |z| < 1\}$; and let

$$f(z) = \frac{a_0}{z} + \sum_{n=1}^{\infty} a_n z^n \quad (a_0 > 0, a_n \geq 0), \tag{1.3}$$

$$f_i(z) = \frac{a_{0,i}}{z} + \sum_{n=1}^{\infty} a_{n,i} z \quad (a_{0,i} > 0, a_{n,i} \geq 0), \tag{1.4}$$

$$g(z) = \frac{b_0}{z} + \sum_{n=1}^{\infty} b_n z^n \quad (b_0 > 0, b_n \geq 0), \tag{1.5}$$

and

$$g_j(z) = \frac{b_{0,j}}{z} + \sum_{n=1}^{\infty} b_{n,j} z^n \quad (b_{0,j} > 0, b_{n,j} \geq 0), \tag{1.6}$$

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be regular and univalent in the punctured disc $U^* = \{z : 0 < |z| < 1\}$.

For a function $f(z)$ defined by (1.3) (with $a_0 = 1$) we define

$$I^0 f(z) = f(z),$$
$$I^1 f(z) = zf^{'}(z) + \frac{2}{z},$$
$$I^2 f(z) = z(I^1 f(z))^{'} + \frac{2}{z}$$

and for $k = 1, 2, 3, \ldots\ldots$

$$I^k f(z) = z\,(I^{k-1} f(z))^{'} + \frac{2}{z}$$
$$= \frac{1}{z} + \sum_{n=1}^{\infty} n^k a_n z^n.$$

The operator I^k was introduced by Frasin and Darus [1].

With the help of the differential operator I^k, we define the classes $\sum S_0^*(k, \alpha, \beta)$ and $\sum C_0(k, \alpha, \beta)$ as follows :

Denote by $\sum S_0^*(k, \alpha, \beta)$, the class of functions $f(z)$ which satisfy the condition

$$\left| \frac{\frac{z(I^k f(z))^{'}}{I^k f(z)} + 1}{\frac{z(I^k f(z))^{'}}{I^k f(z)} + 2\alpha - 1} \right| < \beta \tag{1.7}$$

$(z \in U^*, 0 \le \alpha < 1, 0 < \beta \le 1, k \in N_0)$.

Let $\sum C_0^*(k, \alpha, \beta)$ be the class of functions $f(z)$ for which $-zf^{'}(z) \in \sum S_0^*(k, \alpha, \beta)$.

We note that :

(i) $\sum S_0^*(0, \alpha, \beta) = \sum S_0^*(\alpha, \beta)$, is the class of of meromorphic starlike functions of order α $(0 \le \alpha < 1)$ and type β $(0 < \beta \le 1)$with $a_0 = 1$; studied by Mogra et al. [2];

(ii) $\sum C_0^*(0, \alpha, \beta) = \sum C_0^*(\alpha, \beta)$, is the class of meromorphic convex functions of order $\alpha(0 \le \alpha < 1)$ and typeβ $(0 < \beta \le 1)$ with positive cofficients;

(iii) $\sum S_0^*(k, \alpha, 1) = \sum^*(k, \alpha)$ (Frasin and Darus [1]).

Using similar arguments as given in [1], we can easily prove the following results for functions in the classes $\sum S_0^*(k, \alpha, \beta)$ and $\sum C_0^*(k, \alpha, \beta)$.

A function$f(z) \in \sum S_0^*(k, \alpha, \beta)$ if, and only if,

$$\sum_{n=1}^{\infty} n^k[(1+\beta)n + (2\alpha - 1)\beta + 1]a_n \le 2\beta(1-\alpha)a_0; \tag{1.8}$$

and $f(z) \in \sum C_0^*(k, \alpha, \beta)$ if, and only if,

$$\sum_{n=1}^{\infty} n^{k+1}[(1+\beta)n + (2\alpha - 1)\beta + 1]a_n \le 2\beta(1-\alpha)a_0. \tag{1.9}$$

The quasi-Hadamard product of two or more functions has recently been defined and used by Owa [3–5], Kumar [6–8], Mogra [9,10], Aouf and Darwish [11,12], Hossen [13] and Sekine [14]. Accordingly, the quasi-Hadamard product of two functions $\varphi(z)$ and $\psi(z)$ given by (1.1) and (1.2) is defined by

$$\varphi * \psi(z) = c_1 d_1 z - \sum_{k=2}^{\infty} c_n d_n z^n.$$

Let us define the Hadamard product of two meromorphic univalent functions $f(z)$ and $g(z)$ by

$$f * g(z) = \frac{a_0 b_0}{z} + \sum_{n=1}^{\infty} a_n b_n z^n. \tag{1.10}$$

The Hadamard product of more than two meromorphic functions can similarly be defined.

In [10], Mogra obtained certain results concerning the quasi-Hadamard product of two or more functions in $\sum S_0^*(0, \alpha, \beta) = \sum S_0^*(\alpha, \beta)$ and $\sum C_0^*(0, \alpha, \beta) = \sum C_0^*(\alpha, \beta)$.

In this paper, we introduce the following class of meromorphic univalent functions in U^*.