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Uncertainty measures for rough formulae in rough logic: An axiomatic approach ☆

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ABSTRACT

Rough set theory, initiated by Pawlak, is a mathematical tool in dealing with inexact and incomplete information. Various types of uncertainty measure such as accuracy measure, roughness measure, etc, which aim to quantify the imprecision of a rough set caused by its boundary region, have been extensively studied in the existing literatures. However, a few of these uncertainty measures are explored from the viewpoint of formal rough set theory, which, however, help to develop a kind of graded reasoning model in the framework of rough logic. To solve such a problem, a framework of uncertainty measure for formulae in rough logic is presented in this paper. Unlike the existing literatures, we adopt an axiomatic approach to study the uncertainty measure in rough logic, concretely, we define the notion of rough truth degree by some axioms, such a notion is demonstrated to be adequate for measuring the extent to which any formula is roughly true. Then based on this fundamental notion, the notions of rough accuracy degree, roughness degree for any formula, and rough inclusion degree, rough similarity degree between any two formulae are also proposed. In addition, their properties are investigated in detail. These obtained results will be used to develop an approximate reasoning model in the framework of rough logic from the axiomatic viewpoint.

1. Introduction

Rough set theory [1,2] is proposed to account for the definability of a concept in terms of some elementary ones in an approximation space. It captures and formalizes the basic phenomenon of information granulation. The finer the granulation is, the more concepts are defined in it. For those concepts not definable in an approximation space, the lower and upper approximations can be defined. Recent years have witnessed a wide application of rough set theory in intelligent data analysis, decision making, machine learning and other related fields [3–5].

Since the inception of rough set theory, various types of uncertainty measures such as accuracy measure, roughness measure for rough sets, etc., which aim to quantify the imprecision of a rough set caused by its boundary region, have been extensively studied in the literatures (see [1,2]). For instance, let (U, R) be an approximation space with U being a nonempty finite set and R being an equivalence relation imposed upon U, then for any rough set X, the notion of accuracy measure (always denoted by $\alpha_R(X)$) for X is defined through measuring the ratio of the lower approximation $\underline{R}(X)$ of X in its upper approximation $\bar{R}(X)$, i.e., $\alpha_R(X) = \frac{\underline{R}(X)}{\bar{R}(X)}$. Based upon the fundamental notion, the rough measure of a rough set (denoted by $\rho_R(X)$) is defined $\rho_R(X) = 1 - \alpha_R(X)$, which reflects the intuitive idea that if a rough set is accurate to

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a larger degree, then it is rough to a lower extent, and vice versa. Recently, as the graded version of rough equality, the notion of rough similarity degree between any two rough sets have also been proposed [6]. These proposed notions are not theoretical, but also helpful in knowledge discovery in information tables. However, all the above analyses are restricted to the set theoretical rough set model (crisp or fuzzy), they are seldom explored from the viewpoint of formal rough set theory, which, however, help to develop a kind of approximate reasoning method in the framework of rough logic. To solve this problem, we made a modest attempt in this perspective in [7], wherein we adopt the integrated way to evaluate the rough goodness of formulae and the notion of rough truth degree is thus proposed. By employing various probability measures on the valuation set of rough formula, different types of rough truth degrees can be proposed. In this paper, by abstracting the common properties various types of rough truth degrees share, we propose an axiomatic definition of rough truth degree, that is, we define it by some axioms, then based upon this fundamental notion, the notions of accuracy degree, roughness degree for any formula and rough inclusion degree, the rough similarity degree for any two formulae are also introduced and their properties are examined in detail. The obtained results are not only theoretical, but also helpful in developing a kind of approximate reasoning in the framework of rough logic.

The rest of this paper proceeds as follows: in Section 2, we briefly recall the rough logic PRL initiated by Banerjee in [8–10], which is the central focus of this paper. Then in Section 3, the axiomatic definition of rough truth degree for rough formulae is presented and its properties are examined in detail. Based upon the fundamental concept, the notions of roughness degree and accuracy degree for each formula are defined, respectively, in Section 3. Furthermore, the notions of rough inclusion degree and rough similarity degree are also proposed in Sections 4 and 5, respectively. Lastly, we complete this paper with some concluding remarks, as stated in Section 6.

2. Review of rough set and rough logic

Let's briefly review the basic notions of rough set theory initially proposed by Pawlak [1,2].

Definition 2.1. An approximation space is a tuple $AS = (U, R)$, where U is a non-empty set, also called the universe of discourse, R is an equivalence relation on U, representing indiscernibility at the object level.

Definition 2.2. Let $AS = (U, R)$ be an approximation space defined as above. For any set $X \subseteq U$, if X is a union of some equivalence classes produced by R, then we call X a definable set, and otherwise, a rough set. As for rough set, two definable sets are employed to approximate it from above and from below, respectively. They are

$$\underline{R}(X) = \{x \in U | [x] \subseteq X\}, \tag{1}$$

$$\bar{R}(X) = \{x \in U | [x] \cap X \neq \emptyset\}, \tag{2}$$

where $[x]$ denotes the equivalence block containing x.

Then we call $\bar{R}(X)(\underline{R}(X))$ rough upper (lower) approximation of X. Note that X is a definable set if and only if $\bar{R}(X) = \underline{R}(X)$, and therefore, we also treat definable sets as special cases of rough sets.

Definition 2.3 (*[8–10]*)**.** A structure $\mathscr{P} = (P, \leq, \sqcap, \sqcup, \rightharpoondown, L, \rightarrow, 0, 1)$ is a pre-rough algebra, if and only if

(1) $(P, \leq, \sqcap, \sqcup, \rightarrow, 0, 1)$ is a bounded distributive lattice with least element 0 and largest element 1,
(2) $\rightharpoondown\rightharpoondown a = a$,
(3) $\rightharpoondown (a \sqcup b) = \rightharpoondown a \sqcap \rightharpoondown b$,
(4) $La \leq a$,
(5) $L(a \sqcap b) = La \sqcap Lb$,
(6) $LLa = La$,
(7) $L1 = 1$,
(8) $MLa = La$,
(9) $\rightharpoondown La \sqcup La = 1$,
(10) $L(a \sqcup b) = La \sqcup Lb$,
(11) $La \leq Lb$ and $Ma \leq Mb$ imply $a \leq b$,
(12) $a \rightarrow b = (\rightharpoondown La \sqcup Lb) \sqcap (\rightharpoondown Ma \sqcup Mb)$,
where $\forall a \in P$, $Ma = \rightharpoondown L \rightharpoondown a$.

Example 2.4 (*[8–10]*)**.** Let $\mathbf{3} = (\{0, \frac{1}{2}, 1\}, \leq, \sqcap, \sqcup, \rightharpoondown, L, \rightarrow, 0, 1)$, where $\leq$ is the usual order on real numbers, i.e., $0 \leq \frac{1}{2} \leq 1$, $\sqcap$ and $\sqcup$ are maximum and minimum, respectively. In addition, $\rightharpoondown 0 = 1, \rightharpoondown \frac{1}{2} = \frac{1}{2}, \rightharpoondown 1 = 0, L0 = L\frac{1}{2} = 0, L1 = 1$. Then it can be easily checked that $\mathbf{3}$ is a pre-rough algebra, and also the smallest non-trivial pre-rough algebra.