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Intuitionistic (T, S)-fuzzy CI-algebras

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ABSTRACT

In this paper, we introduce the notion of intuitionistic (T, S)-fuzzy subalgebras in CI-algebras and study their fundamental properties. We get a fuzzy subalgebra from an intuitionistic (T, S)-fuzzy subalgebra. Also the notion of intuitionistic (T, S)-fuzzy (closed) filters of CI-algebras is introduced. We investigate the relationship between intuitionistic (T, S)-fuzzy subalgebras and intuitionistic (T, S)-fuzzy (closed) filters of CI-algebras.

1. Introduction and preliminaries

Imai and Iseki [1] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. BCI-algebras as a class of logical algebras are the algebraic formulations of the set difference together with its properties in set theory and the implicational functor in logical systems. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

Recently, Kim and Kim defined a BE-algebra [2]. Biao Long Meng, defined the notion of CI-algebra as a generalization of a BE-algebra [3]. In [4], Kim studied on this algebra in detail and some fundamental properties of CI-algebras are discussed, and studied in many papers [5–8].

After the concept of fuzzy sets was introduced by Zadeh [9], several studies were conducted on the generalization of the notion of fuzzy sets. The idea of "intuitionistic fuzzy set" was first introduced by Atanassov [10,11], as a generalization of the notion of fuzzy set. The authors studied some fuzzy algebraic structures [12,13].

Motivated by this, in this paper by using t-norm T and s-norm S, we introduce the notion of intuitionistic (T, S)-fuzzy subalgebras of CI-algebras and intuitionistic (T, S)-fuzzy closed filters of CI-algebras.

Now, we rewrite the basic definitions and some elementary aspects that are necessary for the sequel.

Recall that a CI-algebra is an algebra $(X; *, 1)$ of type $(2, 0)$ satisfying the following axioms:

(CI1) $x * x = 1$;
(CI2) $1 * x = x$;
(CI3) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$.

In any CI-algebra X one can define a binary relation "$\leq$" by $x \leq y$ if and only if $x * y = 1$.

A CI-algebra X has the following properties:

(2.1) $y * ((y * x) * x) = 1$,
(2.2) $(x * 1) * (y * 1) = (x * y) * 1$,
(2.3) $1 \leq x \Rightarrow x = 1$ for all $x, y \in X$.

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A non-empty subset S of a *CI*-algebra X is called a subalgebra of X if $x * y \in S$ whenever $x, y \in S$. A mapping $f : X \to Y$ of a *CI*-algebra is called a homomorphism if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

A non-empty subset F of *CI*-algebra X is called a filter of X if (1) $1 \in F$, (2) $x \in F$ and $x * y \in F$ implies $y \in F$. A filter F of *CI*-algebra X is said to be closed if $x \in F$ implies $x * 1 \in F$.

Now, we review some fuzzy logic concepts.

A fuzzy set μ in X, i.e., a mapping $\mu : X \to [0, 1]$, and the complement of μ, denoted by $\bar{\mu}$, is the fuzzy set in X by $\bar{\mu}(x) = 1 - \mu(x)$ for all $x \in X$. For any $\alpha \in [0, 1]$ and a fuzzy set μ in a nonempty set X, the set

$$U(\mu; \alpha) = \{x \in X : \mu(x) \geq \alpha\} \quad (\text{resp. } L(\mu; \alpha) = \{x \in X : \mu(x) \leq \alpha\})$$

is called an upper(resp. lower) level set of μ. Fuzzy set μ is called a fuzzy subalgebra of X if $\mu(x * y) \geq \min(\mu(x), \mu(y))$, for all $x, y \in X$. Note that if μ is a fuzzy subalgebra of a *CI*-algebra X, then $\mu(1) \geq \mu(x)$, for all $x \in X$.

An intuitionistic fuzzy set (briefly, *IFS*) A in a nonempty set X is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$$

where the function $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership and the degree of non-membership, respectively, where

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1$$

for all $x \in X$.

An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$ in X can be identified with an ordered pair (μ_A, γ_A) in $I^X \times I^X$. For the sake of simplicity, we shall use symbol $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$.

Fuzzy set μ is called a fuzzy subalgebra of X with respect to a t-norm T (briefly, a T-fuzzy subalgebra of X) if $\mu(x * y) \geq T(\mu(x), \mu(y))$ for all $x, y \in X$.

Every t-norm T has a useful property: $T(\alpha, \beta) \leq \min(\alpha, \beta)$, for all $\alpha, \beta \in [0, 1]$.

Fuzzy set μ is called a fuzzy subalgebra of X with respect to an s-norm S (briefly, an S-fuzzy subalgebra of X) if $\mu(x * y) \leq S(\mu(x), \mu(y))$ for all $x, y \in X$.

Every s-norm S has a useful property: $S(\alpha, \beta) \geq \max(\alpha, \beta)$, for all $\alpha, \beta \in [0, 1]$.

For a t-norm (or s-norm) P on $[0, 1]$, denote by Δ_P the set of elements $\alpha \in [0, 1]$ such that $P(\alpha, \alpha) = \alpha$, i.e., $\Delta_P := \{\alpha \in [0, 1] | P(\alpha, \alpha) = \alpha\}$.

Definition 1.1 (*[14]*). Let P be a t-norm (or s-norm). A fuzzy set μ in X is said to satisfy the imaginable property with respect to P if $\mathrm{Im}(\mu) \subseteq \Delta_P$.

2. Intuitionistic (T, S)-fuzzy subalgebras of *CI*-algebras

In what follows, let X denote a *CI*-algebra, T be a t-norm and S be an s-norm in $[0, 1]$ unless otherwise specified.

Definition 2.1. Let $A = (\mu_A, \gamma_A)$ be an *IFS* in X. A is called an intuitionistic (T, S)-fuzzy subalgebra of X if

(F1) $\mu_A(x * y) \geq T(\mu_A(x), \mu_A(y))$;
(F2) $\gamma_A(x * y) \leq S(\gamma_A(x), \gamma_A(y))$,

for all $x, y \in X$.

Example 2.2. Let $X := \{1, a, b, c\}$. Define a binary operation "$*$" on X by the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	b
b	1	a	1	a
c	1	1	1	1

Then $(X; *, 0)$ is a *CI*-algebra. Let $T : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by

$$T(\alpha, \beta) = \max(\alpha + \beta - 1, 0)$$

and $S : [0, 1] \times [0, 1] \to [0, 1]$ be a function defined by

$$S(\alpha, \beta) = \min(\alpha + \beta, 1)$$

for all $\alpha, \beta \in [0, 1]$. Then T is a t-norm and S is an s-norm. Define an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ by $\mu_A(1) = \mu_A(b) = \mu_A(c) = 1$, $\mu_A(a) = 0$ and $\gamma_A(1) = \gamma_A(b) = \gamma_A(c) = 0$, $\gamma_A(a) = 1$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic (T, S)-fuzzy subalgebra of X.

Theorem 2.3. *If $\{A_i\}_{i \in I}$ is a family of intuitionistic (T, S)-fuzzy subalgebras of X, then $\bigcap_{i \in I} A_i$ is an intuitionistic (T, S)-fuzzy subalgebra of X, where $\bigcap_{i \in I} A_i = (\vee \mu_i, \wedge \gamma_i)$.*