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A comparison of three gravity inversion methods for crustal thickness modelling in Tibet plateau

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ABSTRACT

Crustal thickness can be determined by gravimetric methods based on different assumptions, e.g. by isostatic hypotheses. Here we compare three gravimetric inversion methods to estimate the Moho depth. Two Moho models based on the Vening Meinesz–Moritz hypothesis and one by using Parker–Oldenburg's algorithm, which are investigated in Tibet plateau. The results are compared with CRUST2.0, and it will be presented that the estimated Moho depths from the Vening Meinesz–Moritz model will be better than the Parker–Oldenburg's algorithm.

1. Introduction

The *Mohorovičić discontinuity,* usually called the *Moho*, is the boundary between the Earth's crust and mantle. Different methods can be mentioned to estimate the Moho depth: seismic method such as CRUST2.0 (Bassin et al., 2000) and gravimetric–isostatic methods. According to the isostatic hypothesis a mountain is compensated by a mass deficiency beneath it, where the crust is floating on the viscous mantle. In addition different gravimetric–isostatic hypotheses exist for estimating the crustal thickness/Moho and density of the Earth's crust, and it is not clarified which one is the most suitable to use in geophysical and geodynamical applications. There are two classical isostatic models for topographic mass compensation: (a) Pratt's (1855) and (b) Airy's (1855). According to Pratt, the mass of each topographic column of the same cross-section is equal above the level of compensation depth. According to Airy, mountains are floating on the mantle with higher density, and mountains have roots that construct the compensation. Both hypotheses are highly idealized when they assume the topographic mass compensation to be strictly local. Vening Meinesz (1931) modified Airy's hypothesis by introducing regional instead of local compensation. Parker's (1972) model was based on the relation between the vertical gravity effect and its causative topographic mass in the Fourier domain. The Parker model was constructed based on variable Moho depth and the constant density contrast. This model is close to that of Vening Meinesz from the concept point of view. Oldenburg (1974) deduced a method to compute the density contrast of crust and mantle from the gravity anomaly in a Cartesian coordinate system from Parker's method. Also he defined a method to stabilize the inversion in Parker's method. Several authors presented a variety of methods to compute the geometry of a density interface related to known gravity anomalies, see e.g. Cordell and Henderson (1968) and Dyrelius and Vogel (1972) that used the technique presented by Parker (1972). The Parker–Oldenburg (PO) method used by Gómez Oritz and Agarwal (2005) and Shin et al. (2006) to estimate the Moho depth in Brittany (France) and Ulleung Basin (South of Korea), respectively, based on Fast Fourier Transform (FFT) technique. Shin et al. (2007) studied the Moho undulations beneath Tibet from the GRACE (Gravity Recovery and Climate Experiment, Tapley et al., 2005) gravity data based on PO's method. Antarctica crustal thickness investigated by Block et al. (2009) from satellite gravity data by using PO's method, too. They used global gravity field from GRACE to estimate the Moho depth through gravity inversion.

Until now we have tried to review the prior studies on the Moho depth estimation. Now a different gravimetric–isostatic model is presented. Moritz (1990, Section 8) presented the inverse isostatic problem based on the Vening Meinesz hypothesis in a global spherical approximation, and we hereafter call this method the Vening Meinesz–Moritz (VMM) problem/method. Recently Sjöberg (2009) formulated this problem by solving a non-linear Fredholm equation of the first kind, and he has presented some approximate and practical methods to estimate the crustal thickness by the gravimetric data.

The main applications of the Moho models can be mentioned such as forward dynamic modelling, numerical heat flow

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calculations and seismologic applications. The crustal thickness can be used for downward continuation of the satellite data such as Gravity field and steady-state Ocean Circulation Explorer (GOCE) to recover the gravity anomaly. In this method the topographic–isostatic potential can be determined by the Moho depth model then their effect will removed from the satellite data for smoothing the satellite data (Eshagh and Bagherbandi, 2011). This is one of the geodetic example for the Moho depth. Also the Moho depth model can be applied for determination of the Moho density contrast (Sjöberg and Bagherbandi, 2011). The crustal thickness estimated by the VMM model can be applied to construct a synthetic Earth gravity model (SEGM), by the topographic–isostatic coefficients (Pavlis and Rapp, 1990; Haagmans, 2000). The main motivation to use the topographic-isostatic harmonic coefficients is to create the SEGM, is the similarity of power spectra of the topographic-isostatic and Earth gravitational models such as EGM08 (Pavlis et al., 2004, 2008) and also large correlation between them in medium/high-wavelength. The SEGM describes the potential field of the synthetic Earth. It must be realistic and consistent with the Earth's topography. To achieve this, an existing global geopotential model is used to describe the low degrees, whereas the medium and high degrees are obtained from a global topographic-isostatically induced potential (Haagmans, 2000). However, the method is rather *simple*, *easily* applicable and it can provide a good starting point for the *validation* of various gravity field recovery techniques.

In this study three inversion methods are investigated to estimate the Moho depth. All these models use the same assumption to estimate the Moho depth. On the other hand, these models have the same mathematical nature for recovery of Moho from gravity anomalies. We will compare difference gravimetric models in comparison with CRSUT2.0. Advantage and disadvantage of the Moho Models will be discussed. In addition, we know that the inversion methods are an improperly posed problem, and sensitive to all kinds of errors such as discretization error of the integral equation. In this paper we will discuss about stabilization of each inversion methods for estimating the Moho depth.

2. The Vening Meinesz–Moritz isostatic hypothesis

The Vening Meinesz–Moritz problem (Vening Meinesz, 1931; Moritz, 1990) is to determine the Moho depth $T(P)$ such that the compensating attraction $A_C(P)$ totally compensates the Bouguer gravity anomaly $\Delta g_B(P)$ on the Earth's surface, implying that the isostatic anomaly $\Delta g_I(P)$ vanishes for point P on the Earth's surface (Sjöberg, 2009):

$$\Delta g_I(P) = \Delta g_B(P) + A_C(P) = 0. \tag{1}$$

Eq. (1) is the fundamental equation for determining the Moho depth isostatically. It should be stated that the compensation attraction is a function of the Moho depth as well as the position of point P. In order to obtain the mathematical expression of $A_C(P)$, consider the compensation potential at an arbitrary point P based on the Newton integral:

$$V_C(P) = G\Delta\rho \iint_\sigma \int_{R-T}^{R-T_0} \frac{r_Q^2\, dr_Q}{L_{PQ}} d\sigma, \tag{2}$$

where $\Delta\rho$ is the density contrast between crust and mantle, G is the Newtonian gravitational constant, R is the radius of a sphere equal to semi-major axis of the reference ellipsoid, T the variable Moho depth, T_0 is the normal Moho depth or the global mean depth of Moho; see Sjöberg (2009) for its mathematical expression. σ is the unit sphere and $L_{PQ} = \sqrt{r_P^2 + r_Q^2 - 2r_Q r_P \cos\psi}$, where r_P and r_Q are the geocentric distances to the computation point P and integration point Q, respectively, and ψ is the geocentric angle between these two points. Eq. (2) can also be written by:

$$V_C(P) = V_{C_0}(P) + dV_C(P), \tag{3a}$$

where

$$V_{C_0}(P) = G\Delta\rho \iint_\sigma \int_R^{R-T_0} \frac{r_Q^2\, dr_Q}{L_{PQ}} d\sigma \tag{3b}$$

is the potential contribution from the mean Moho depth and

$$dV_C(P) = G\Delta\rho \iint_\sigma \int_{R-T}^{R} \frac{r_Q^2\, dr_Q}{L_{PQ}} d\sigma \tag{3c}$$

is the contribution from the variable Moho depth T. The compensation attraction at point P can thus be obtained by taking radial derivative of Eq. (3a) and inserting it into Eq. (1) can be written:

$$\Delta g_B(P) + A_{C_0}(P) + dA_C(P) = 0. \tag{4a}$$

where $A_{C0}(P)$ and $dA_C(P)$ are the compensation attractions of the mean and variable Moho, respectively. Here the Moho depth T is implicitly hidden in the integral of $dA_C(P)$. Rearrangement of Eq. (4a) yields:

$$dA_C(P) = -[\Delta g_B(P) + A_{C_0}(P)], \tag{4b}$$

and by expanding the first term of Eq. (3b) in a series of Legendre's polynomials, and performing the integration with respect to the radius r and using the binomial expansion, one can obtain the normal compensation attraction $A_{C_0}(P)$ as (Sjöberg, 2009):

$$\widetilde{A}_{C_0}(P) = (A_{C_0})_{r_P=R} = \frac{4\pi kR}{3}[(1-\tau_0)^3 - 1] \approx -4\pi k T_0. \tag{4c}$$

By substituting the first radial derivative of Eq. (3c) into Eq. (4b) the Moho depth can be recovered by solving the following integral Fredholm equation of first kind (Sjöberg, 2009):

$$R \iint_\sigma K(r_P, \psi, s) d\sigma = f(P), \tag{5a}$$

where

$$f(P) = -[\Delta g_B(P) + A_{C_0}(P)]/(G\Delta\rho), \tag{5b}$$

$$K(r_P, \psi, s) = \sum_{n=0}^{\infty} \frac{n+1}{n+3} \left(\frac{R}{r_P}\right)^{n+2} (1 - s^{n+3}) P_n(\cos\psi), \tag{5c}$$

$$s = 1 - \tau \text{ and } \tau = \frac{T}{R}. \tag{5d}$$

where $P_n(\cos\psi)$ is the Legendre polynomial of degree n. In fact the residual compensation attraction $dA_C(P)$ is equal to the left side of Eq. (5a). Eq. (5a) is the main equation of inverse problem in isostasy.

Eq. (5c) can be written by the following closed form formula (see Appendix A for a proof):

$$K(r_P, \psi, s) = \frac{a_1^2}{l_{a_1}} - \frac{2}{a_1} F_1(P) - s\frac{a_2^2}{l_{a_2}} + \frac{2s}{a_2} F_2(P), \tag{5e}$$

where

$$F_i(P) = \frac{1}{2}\left[(a_i + 3t)l_{a_i} - 3t + (3t^2 - 1)\ln\frac{(a_i - t + l_{a_i})}{(1-t)}\right],$$
$i = 1$ or 2.

$$l_{a_i} = \sqrt{1 + a_i^2 - 2a_i t}, \quad i = 1 \text{ or } 2. \tag{5g}$$

where $a_1 = \frac{R}{r_P}$, $a_2 = \frac{Rs}{r_P}$ and $t = \cos\psi$.