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Positive solutions for a coupled system of nonlinear fractional differential equations with integral boundary conditions

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ABSTRACT

In the this paper, we establish sufficient conditions for the existence and nonexistence of positive solutions to a general class of integral boundary value problems for a coupled system of fractional differential equations. The differential operator is taken in the Riemann–Liouville sense. Our analysis rely on Banach fixed point theorem, nonlinear differentiation of Leray–Schauder type and the fixed point theorems of cone expansion and compression of norm type. As applications, some examples are also provided to illustrate our main results.

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1. Introduction

Fractional differential equations arise in many engineering and scientific disciplines as the mathematical modelling of systems and processes in the fields of physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. involves derivatives of fractional order. Fractional differential equations also serve as an excellent tool for the description of hereditary properties of various materials and processes. In consequence, the subject of fractional differential equations is gaining much importance and attention. There are a large number of papers dealing with the existence or multiplicity of solutions or positive solutions of initial or boundary value problem for some nonlinear fractional differential equations. For details and examples, see [1–8] and the references therein. In [9–11], the authors have discussed the existence of positive solutions for boundary value problem of nonlinear fractional differential equations. In [12], Feng et al. studied the existence and multiplicity of positive solutions for the following higher-order singular boundary value problem of fractional differential equation:

$$\begin{cases} D^{\alpha}u(t) + g(t)f(t, u(t)) = 0, \quad 0 < t < 1, \\ u(0) = u'(0) = \cdots = u^{(n-2)}(0) = 0, u(1) = \displaystyle\int_0^1 h(t)x(t)dt, \end{cases}$$

where D is the standard Riemann–Liouville fractional derivative of order $n-1 < \alpha \le n, n \ge 3, g \in C((0,1), [0,+\infty))$ and g may be singular at $t = 0$ or / and at $t = 1$, $h \in L^1[0, 1]$ is nonnegative, and $f \in C([0, 1] \times [0, +\infty), [0, +\infty))$.

Recently, many people have established the existence and uniqueness for solutions of some systems of nonlinear fractional differential equations, readers can see [13–20] and references cited therein. For example, Su [21] established sufficient conditions for the existence of solutions for a two-point boundary value problem for a coupled system of fractional

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differential equations:

$$\begin{cases} D^{\alpha}u(t) = f(t, v(t), D^{\mu}v(t)), D^{\beta}v(t) = f(t, u(t), D^{\nu}u(t)), & 0 < t < 1, \\ u(0) = u(1) = v(0) = v(1) = 0, \end{cases}$$

where $1 < \alpha, \beta < 2, \mu, \nu > 0, \alpha - \nu \geq 1, \beta - \mu \geq 1, f, g: [0, 1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are given functions, and D is the standard Riemann–Liouville fractional derivative. Ahmad and Nieto [22] extended the results of [21] to a three-point boundary value problem for the following coupled system of fractional differential equations:

$$\begin{cases} D^{\alpha}u(t) = f(t, v(t), D^{\mu}v(t)), D^{\beta}v(t) = f(t, u(t), D^{\nu}u(t)), & 0 < t < 1, \\ u(0) = 0, u(1) = \gamma u(\eta), v(0) = 0, v(1) = \gamma v(\eta), \end{cases}$$

where $1 < \alpha, \beta < 2, \mu, \nu, \gamma > 0, 0 < \eta < 1, \alpha - \nu \geq 1, \beta - \mu \geq 1, \gamma\eta^{\alpha-1} < 1, \gamma\eta^{\beta-1} < 1, f, g: [0, 1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are given continuous functions, and D is the standard Riemann–Liouville fractional derivative. Wang et al. [23] obtained the existence and uniqueness of positive solution to nonzero boundary values problem for a coupled system of nonlinear fractional differential equations:

$$\begin{cases} D^{\alpha}u(t) = f(t, v(t)), D^{\beta}v(t) = f(t, u(t)), & 0 < t < 1, \\ u(0) = 0, u(1) = au(\xi), v(0) = 0, v(1) = bv(\xi), \end{cases}$$

where $1 < \alpha, \beta < 2, 0 \leq a, b \leq 1, 0 < \xi < 1, f, g: [0, 1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are given functions, and D is the standard Riemann–Liouville fractional derivative.

Motivated by the above mentioned works, we consider the existence and nonexistence of positive solutions to boundary values problem for a coupled system of nonlinear fractional differential equations as follows:

$$\begin{cases} D^{\alpha}u(t) + a(t)f(t, v(t)) = 0, D^{\beta}v(t) + b(t)g(t, u(t)) = 0, & 0 < t < 1, \\ u(0) = 0, u(1) = \displaystyle\int_0^1 \phi(t)u(t)dt, v(0) = 0, v(1) = \int_0^1 \psi(t)v(t)dt, \end{cases} \tag{1.1}$$

where $1 < \alpha, \beta \leq 2$, $a, b \in C((0, 1), [0, +\infty))$, $\phi, \psi \in L^1[0, 1]$ are nonnegative and $f, g \in C([0, 1] \times [0, +\infty), [0, +\infty))$, and D is the standard Riemann–Liouville fractional derivative. By applying Banach fixed point theorem, nonlinear differentiation of Leray–Schauder type and the fixed point theorems of cone expansion and compression of norm type, sufficient conditions for the existence and nonexistence of positive solutions to a general class of integral boundary value problems for a coupled system of fractional differential equations are obtained. Furthermore, some example are also provided to illustrate our main results.

2. Preliminaries

In this section, we introduce definitions and preliminary facts which are used throughout this paper.

Definition 2.1 (*See [24,25]*)**.** The fractional integral of order q with the lower limit a for a function f is defined as

$$I_{a+}^{q}f(t) = \frac{1}{\Gamma(q)} \int_a^t (t-s)^{q-1}f(s)ds, \quad t > a, q > 0, \tag{2.1}$$

provided the right-hand side is pointwise defined on $[a, \infty)$, where $g \in C[a, b]$ and Γ is the gamma function. For $a = 0$, the fractional integral (2.1) can be written as $I_{0+}^{\alpha}h(t) = h(t) * \varphi_{\alpha}(t)$, where $\varphi_{\alpha}(t) = t^{\alpha-1}/\Gamma(\alpha)$ for $t > 0$ and $\varphi_{\alpha}(t) = 0$ for $t \leq 0$.

Definition 2.2 (*See [24,25]*)**.** Riemann–Liouville derivative of order q with the lower limit $t0$ for a function $f: [a, \infty) \to \mathbb{R}$ can be written as

$$D_{a+}^{q}f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_a^t (t-s)^{n-q-1}f(s)ds, \quad t > a, n-1 < q < n.$$

Lemma 2.3 (*Nonlinear Differentiation of Leray–Schauder Type, See [26]*)**.** *Let E be a Banach space with $C \subseteq E$ closed and convex. Let U be a relatively open subset of C with $0 \in U$ and let $T: U \to C$ be a continuous and compact mapping. Then either*

(a) *the mapping T has a fixed point in U, or*
(b) *there exist $u \in \partial U$ and $\lambda \in (0, 1)$ with $u = \lambda Tu$.*

Lemma 2.4 (*Fixed-Point Theorem of Cone Expansion and Compression of Norm Type, See [27]*)**.** *Let P be a cone of real Banach space E, and let Ω_1 and Ω_2 be two bounded open sets in E such that $0 \in \Omega_1 \subset \overline{\Omega}_1 \subset \Omega_2$. Let operator $A : P \cap (\overline{\Omega}_2 \setminus \Omega_1) \to P$ be completely continuous operator. Suppose that one of the two conditions holds:*

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