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Shape optimization for tumor location

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ABSTRACT

In non-invasive thermal diagnostics, accurate correlations between the thermal image at skin surface and interior human physiology are desired. In this work, an estimation methodology to determine unknown geometrical parameters of an embedded tumor is proposed. We define a functional that represents the mismatch between a measured experimental temperature profile, which may be obtained by infrared thermography on the skin surface, and the solution of an appropriate boundary problem. This functional is related to the geometrical parameters through the solution of the boundary problem, in such a way that finding the minimum of this functional form also means finding the unknown geometrical parameters of the embedded tumor. Sensitivity analysis techniques coupled with the adjoint method were considered to compute the shape derivative of the functional. Then, a nonmonotone spectral projected gradient method was implemented to solve the optimization problem of finding the optimal geometric parameters.

1. Introduction

It is well known that body temperature is an indicator of health. In general, the body surface temperature is controlled by the blood circulation underneath the skin, local metabolism, and the heat exchange between the skin and its environment [1–3]. Changes in any of these parameters can induce variations in temperature and heat flux at the skin surface reflecting the physiological state of the human body. The particular tumor architecture and angiogenesis processes can lead to an abnormal situation. Inflammation, metabolic rate, interstitial hypertension, abnormal vessel morphology and lack of response to homeostatic signals are some of the particular features that make tumors to behave differently than normal tissue in terms of heat production and dissipation. Skin temperature above a breast tumor or a malignant melanoma, a tumor of melanocytes which are found predominantly in skin, has been found to be several degrees higher than that of the surrounding area [4–7].

Therefore, the abnormal temperature on the skin surface can be used in order to predict the location and size of an embedded tumor as well as to study the tumor evolution after a treatment procedure.

Medical infrared thermography is a non-invasive and non-contact functional imaging method that measures the radiation emitted from the skin surface and provides information about subtle temperature changes in it. Medical applications of infrared thermography are not a recent phenomenon. However, in the past years their success was rather limited mainly due to the complexity, high cost, and poor sensitivity provided by the generation of infrared cameras that were available at that time. Nowadays, advances in infrared technology have again promoted its medical application as a promising non-invasive tool for imaging the functionality of superficial layers of tissues and the influence of vascular, neurogenic and metabolic processes that affect them. In [7], Santa Cruz et al. have investigated the correlation, by means of thermography, in patients treated with boron neutron capture therapy (BNCT), between the spatial extension of the acute skin reaction and the superficial dose distribution, in order to determine tolerance doses and therefore to optimize the BNCT treatment. They have also concluded that given the ability of thermography to observe the functional aspects of tissues, the

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technique can help to locate abnormally high temperature regions as well as melanoma nodules that are virtually invisible in CT images.

The objective of this paper is the development of a methodology to estimate the location and size of an embedded (malignant melanoma) tumor region with the help of abnormal temperature profiles measured on the skin surface. We have defined a functional that represents the mismatch between a measured experimental temperature profile at the skin surface and the solution of the Pennes equation [8] with appropriate boundary conditions. This bioheat equation describes the heat transfer inside living tissues and it is widely used to solve the temperature distribution for thermal therapy [9–12]. According to literature [4–7] healthy and tumor regions have different thermal coefficient values and heat sources. We have therefore assumed that all the thermal coefficients and heat sources are piecewise constant functions. In this way we can view the location problem as a problem of finding the shape of a subdomain $\omega \subset \Omega$, using the knowledge of its (constant) density and the measurement data in some open subset of the boundary $\partial\Omega$, where Ω is the domain where the Pennes equation is considered. The optimal shape is the one that minimize the objective functional defined, and in order to find this minimum we have employed some results from the shape sensitivity analysis theory.

After we have introduced the medical facts about the skin cancer and the relation to the body temperature, the plan for the rest of this work is as follows: In Section 2 we describe the mathematical model proposed to simulate the heat transfer in a human body in 2D domains. The following section is devoted to state the inverse problem and define the functional to be minimized. Then, in Section 4 we present an introduction to the theory of sensitivity analysis and shape form derivative. In Section 5 we present the computations of the shape form derivative that allows us to compute the minimum of the functional defined in Section 3. After that, in Section 6 the results obtained from simulations with and without random noise are exposed. Moreover we compare these results with the simulations obtained using another optimization algorithm, [13]. Finally, in Section 7 some comments and conclusions are given and an Appendix is also present.

2. Mathematical model

A number of bioheat transfer equations for living tissues have been proposed since the landmark paper by Pennes appeared in 1948, [8]. The main theoretical contribution is the suggestion that the rate of heat transfer between the blood and tissue is proportional to the product of the volumetric perfusion rate and to the difference between the arterial blood temperature, and the local tissue temperature. The relationship was expressed as follows:

$$h_b = G_b \rho_b c_b (1 - \kappa)(T_b - u)$$

where h_b is the rate of heat transfer per unit volume of tissue, G_b is the perfusion rate per unit volume of tissue, ρ_b is the density of blood, c_b is the specific heat of blood, κ is a factor that accounts for incomplete thermal equilibrium between blood and tissue, T_b is the temperature of arterial blood, and u is the local tissue temperature. Although at first κ can take values in the interval $0 < \kappa < 1$, Pennes, in [8], as in most of the works related, set $\kappa = 0$. Also, he considered arterial temperature constant and equal to the body core temperature.

The proposed equation also includes a term that represents the heat transfer by conduction through the tissue and a term that represents the volumetric metabolic heat generation. These two terms coupled with the term that represents the heat transfer due to the circulation of blood give rise to the following steady-state Pennes equation:

$$-\operatorname{div}(\sigma(x)\nabla u(x)) + k(x)(u(x) - T_b) = q(x), \quad x \in \Omega \subset \mathbb{R}^n,\ n = 2, 3 \tag{1}$$

where σ is the thermal conductivity, $k = G_b \rho_b c_b$ is the perfusion coefficient, q is the metabolic heat source and T_b is the constant blood temperature. For convenience from now on we will denote $Q = q + kT_b$.

Using the fact that the thermal conductivity, the perfusion and the metabolic activity in a melanoma tumor is significantly higher than in normal tissue, we have considered that all these coefficients are piecewise continuous. For example, we can define the thermal conductivity by:

$$\sigma(x) = \begin{cases} \sigma_1, & \text{if } x \in \Omega - \overline{\omega}, \\ \sigma_0, & \text{if } x \in \omega, \end{cases} \tag{2}$$

where ω represents the tumor region and $\Omega - \overline{\omega}$ the healthy tissue (see Fig. 1). Then, if we define $u_1 = u_{|\Omega - \overline{\omega}}$ and $u_0 = u_{|\omega}$ we arrive to the following transmission problem:

$$(P) \quad \begin{cases} -\sigma_1 \Delta u_1 + k_1 u_1 = Q_1, & \text{in } \Omega - \overline{\omega}, \\ -\sigma_0 \Delta u_0 + k_0 u_0 = Q_0, & \text{in } \omega, \\ u_1 = u_0, & \text{on } \partial\omega, \\ -\sigma_1 \dfrac{\partial u_1}{\partial \eta} = -\sigma_0 \dfrac{\partial u_0}{\partial \eta}, & \text{on } \partial\omega, \\ -\sigma_1 \dfrac{\partial u_1}{\partial \eta} = \alpha(u_1 - T_a), & \text{on } \Gamma_u, \\ -\sigma_1 \dfrac{\partial u_1}{\partial \eta} = 0, & \text{on } \Gamma_l, \\ u_1 = T_b, & \text{on } \Gamma_b, \end{cases} \tag{3}$$