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Finite strain estimation from deformed elliptical markers: The minimized $\overline{Ri}$ (MIRi) iterative method

Stefano Vitale

Dipartimento di Scienze della Terra, dell'Ambiente e delle Risorse (DiSTAR), Università di Napoli Federico II, Largo San Marcellino 10, 80138 Napoli, Italy

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ABSTRACT

A new technique for estimating the finite strain of deformed elliptical markers is presented. This method is based on the property of the arithmetic mean $\overline{Rf}$ of the deformed object aspect ratios Rf to reach its minimum value in the undeformed state when they correspond to the initial aspect ratios Ri. The minimized $\overline{Ri}$ (MIRi) iterative method furnishes the best results when, in the pre-strain state, the markers are uniformly orientated for every aspect ratio (Ri) class. A Matlab code, provided in this study, finds the best values of strain Rs and maximum stretching direction X that minimize the arithmetic mean $\overline{Ri}$ by means of several iterations. In order to define the uncertainties of Rs and X, the code: (i) re-samples h-times the original (Ri, θ) dataset; (ii) assigns random values to the initial long axis angles θ; (iii) deforms newly the synthetic dataset; (iv) re-applies the MIRi method; and finally (v) estimates the standard deviation for the (Rs, X) values. Tests of the method on synthetic aggregates of elliptical markers and two naturally deformed rocks provide strain values that are compared with estimations from other available methods.

1. Introduction

The evaluation of finite strain in rocks is central to understanding many aspects of natural rock deformation including strain magnitude, symmetry, orientation and distribution at all scales (e.g. Talbot and Sokoutis, 1995; Yonkee, 2005; Alsleben et al., 2008; Vitale and Mazzoli, 2008, 2009; Sarkarinejad et al., 2010; Dasgupta et al., 2012; Zhang et al., 2013). Several studies have devised powerful tools for determining finite strain by means of algebraic and geometrical approaches (e.g. Ramsay, 1967; Dunnet, 1969; Elliott, 1970; Dunnet and Siddans, 1971; Matthews et al., 1974; Shimamoto and Ikeda, 1976; Lisle, 1977a,b; 1985; Fry, 1979; Erslev, 1988; Mulchrone et al., 2003; Yamaji, 2005, 2008; Vitale and Mazzoli, 2010; Shan and Liang, 2014).

Many rocks, to a first order, can be considered as aggregates of different grains with different sizes and shapes that are differentiated as "matrix and inclusions". The matrix comprises grains with a smaller size within which occur "inclusions" defined by the coarser grains. The matrix and inclusions can show different rheological behaviors, a reflection of different viscosities, wherein each exhibits different finite strain relative to an imposed bulk strain. In turn, the effective matrix-inclusion viscosity contrast depends also on the relative concentrations of matrix and inclusions (Gay, 1968; Treagus and Treagus, 2001; Vitale and Mazzoli, 2005) with viscosity contrast approaching unity for low matrix concentrations (such as grain-supported rocks). When this circumstance is verified, or when matrix and inclusions have the same viscosity (i.e. there is no viscosity contrast), the bulk strain Rs corresponds to that calculated from the inclusions. Several methods exist, allowing one to estimate finite bulk strain (Rs) from deformed markers on a plane (2D strain analysis). These can be grouped into three main classes: (i) graphical, (ii) iterative, and (iii) algebraic; that are summarized below. Geometric relationships between strain elements described in this paper are shown in Fig. 1; all abbreviations are listed, with relevant captions, in Table 1.

Graphical methods, providing approximations of Rs and direction of the maximum stretching X, include the Rf/ϕ method (Ramsay, 1967; Dunnet, 1969; Lisle, 1985). This technique works in the case of homogeneously and passively deformed markers without an initial preferred orientation. The (Rf^j, ϕ^j) data, where Rf^j is the aspect ratio and ϕ^j the orientation of the long axis of the j-th deformed object, are compared with sets of pre-strain state aspect ratio (Ri) curves. The center of these "onion" curves, that fits best the data distribution, provides the expected strain Rs and direction X. However the choice of the best-fit curves is arbitrary. Another popular technique is the Fry method (Fry, 1979). Starting from the centroid coordinates of every marker, the bulk strain is calculated

E-mail address: stefano.vitale@unina.it.

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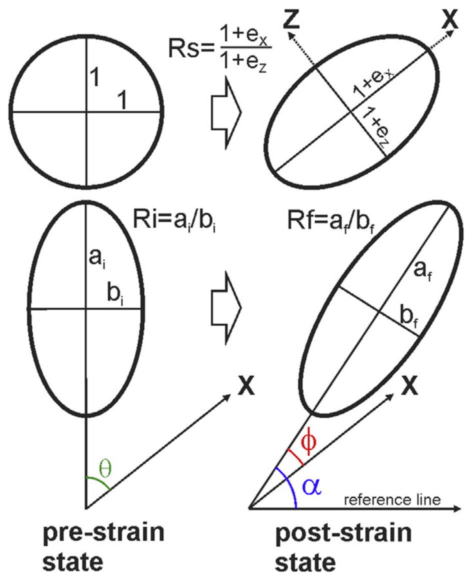

Fig. 1. Geometric relationships between strain features described in the text.

regardless the initial marker shape, with the assumptions that the undeformed objects are uniformly oriented and have about the same area. The latter constraint is overcome in the normalized Fry method (Erslev, 1988) where every marker area is normalized.

Iterative methods include the θ-curve technique (Lisle, 1977b; Peach and Lisle, 1979), which consists of finding the best uniform distribution of (Rf,ϕ) data with respect the θ-curves calculated using the χ^2 test, where the θ^j is the angle that the long axis of every marker forms with the X-direction in the pre-strain state. The method assumes that the distribution of the undeformed marker orientation was uniform. However this method does not directly estimate the direction of the finite strain ellipse X-axis, corresponding to the symmetry line of the (Rf,ϕ) data that has to be previously established. Borradaile (1976) proposed a method in which the elliptical marker directions are undeformed several times with different values of incremental strain and compared with a random array of directions with the help of the χ^2 test. In common with many other methods is the assumption that the pre-deformation orientation distribution was uniform.

Of the methods based on algebraic analyses, some calculate only the bulk strain Rs. These include estimation using the arithmetic mean of the aspect ratios (Hossack, 1968) that supposes an initial spherical shape of the strain markers and determination of the harmonic mean $\overline{Rf}_H$ (Lisle, 1977a) that requires a uniform distribution of orientations for each shape fraction. Matthews et al. (1974) provide a statistic approach for determining Rs, but the maximum stretching direction X has to be assumed. Another algebraic method able to determine both Rs and X is that proposed by Shimamoto and Ikeda (1976). Here, the basic assumptions are that the markers, in the undeformed state, are (i) passively deformed; (ii) characterized by elliptical shapes, and (iii) randomly orientated. Mulchrone et al. (2003) furnish (Rs, X) estimations by an algebraic method and uncertainties using the bootstrap technique assuming that: (i) the long axis orientation of ellipsoidal markers is a uniform random variable; (ii) the distribution of axial ratios in pre-strain state is independent of orientation; (iii) markers are passively deformed. Yamaji (2005) furnishes an inverse method for deformed elliptical objects also in the case of the occurrence of a pre-strain fabric characterized by a class of anisotropy. Finally Yamaji (2008, 2013) has proposed an unified theory of 2D strain analysis consisting in the application of the hyperbolic vector mean method which results are comparable with those of the methods of Mulchrone et al. (2003) and Shimamoto and Ikeda (1976).

In this paper, a further method is presented that is able to calculate approximations of Rs (bulk strain), direction of maximum stretching direction X and relative uncertainties. This method is a corollary of the existing theory of strain analysis and can be used as an additional estimation to compare with other approximations resulting from the established methods.

Table 1
Abbreviations used in the text.

	Notation
X	Long axis of finite strain ellipse (maximum stretching direction)
Z	Short axis of finite strain ellipse
e_X	Extension along the X direction
e_Z	Extension along the Z direction
Rs	Finite strain
a_i	Long axis of the undeformed marker
b_i	Short axis of the undeformed marker
Ri	Aspect ratio of the undeformed marker
θ	Angle between X and the long axis of the undeformed marker
a_f	Long axis of the deformed marker
b_f	Short axis of the deformed marker
Rf	Aspect ratio of deformed marker
α	Angle between the horizontal line and the long axis of the deformed marker
ϕ	Angle between X and the long axis of the deformed marker
$\overline{Rf}_H$	Harmonic mean of Rf
$\overline{Ri}$	Arithmetic mean of Ri

2. The minimized $\overline{Ri}$ (MIRi) iteration method

In order to obtain best estimations of bulk strain Rs and maximum stretching direction X within a plane, the MIRi method assumes that:

1. deformation is homogeneous within a relevant volume element;
2. pre-strain state inclusions can be approximated to ellipses (e.g. Mulchrone and Roy Choudhury, 2004);
3. the distribution of long axis orientation is uniform for each ellipticity class.

Point 3 implies that the axial ratios in the pre-strain state are independent of orientation. Another condition necessary to estimate the bulk strain from deformed inclusions is that the markers are passively deformed. In the case the viscosity contrast between inclusions and matrix is different from unity, the calculated strain Rs does not match with the bulk strain, providing only an estimation of the inclusion strain (e.g. Treagus and Treagus, 2001).

According to Lisle (1977a), measurements of the aspect ratios Rf^j of every j-th deformed object do not allow the direct calculation of the exact value of the object strain Rs from an algorithm, but only approximations of the real value. For example, by comparing the arithmetic ($\overline{Rf}$) and harmonic mean ($\overline{Rf}_H$) of the aspect ratios Rf^j for elliptical markers with the same initial aspect ratio Ri^j and a uniform orientation distribution (Fig. 2), Lisle (1977a) shown as the $\overline{Rf}_H$ furnishes the best approximation of Rs, on the contrary the arithmetic mean $\overline{Rf}$ provides the worst estimation (Fig. 2). This is true for all elliptical inclusions regardless of their initial aspect ratio Ri^j. The harmonic mean is a sufficient approximation for low values of the initial aspect ratio (Ri^j between 1 and 2; Fig. 2), whereas for high initial aspect ratios and low values of the strain, the difference between the harmonic mean and the true strain can be very large;