Computers and Mathematics with Applications 62 (2011) 2126–2131

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Some new solitonary solutions of the modified Benjamin–Bona–Mahony equation

Muhammad Aslam Noor [a,b], Khalida Inayat Noor [a], Asif Waheed [a,*], Eisa A. Al-Said [b]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Park Road, Islamabad, Pakistan

[b] Mathematics Department, College of Science, King Saud University, Riyadh, Saudi Arabia

ARTICLE INFO

Article history:
Received 15 December 2010
Received in revised form 28 June 2011
Accepted 28 June 2011

Keywords:
Exp-function method
Soliton solutions
Nonlinear evolution equations

ABSTRACT

In this paper, we use the exp-function method to construct some new soliton solutions of the Benjamin–Bona–Mahony and modified Benjamin–Bona–Mahony equations. These equations have important and fundamental applications in mathematical physics and engineering sciences. The exp-function method is used to find the soliton solution of a wide class of nonlinear evolution equations with symbolic computation. This method provides the concise and straightforward solution in a very easier way. The results obtained in this paper can be viewed as a refinement and improvement of the previously known results.

1. Introduction

Many phenomena in engineering and applied sciences are modeled by nonlinear evolution equations. Solitonary solutions of nonlinear evolution equations provide better understanding of the physical mechanism of phenomena. Nonlinear evolution equations also characterize the wave phenomena in fluid dynamics, hydro magnetic waves in cold plasma, acoustic waves in crystals, elastic media, optical fiber and some other branches of engineering and applied sciences; see [1–20] and the references therein. A substantial amount of work has been invested for solving such models. Several analytical techniques for solving nonlinear evolution equations have been presented, such as the inverse scattering method, the perturbation method, the sine–cosine method, the homotopy perturbation method, Backlund transformation, Hirota's method, Darboux transformation, Painleve expansions, extended tanh-function, the F-expansion method, the extended F-expansion method and so on.

In this paper, we consider a well-known nonlinear evolution equation which is also known as the generalized Benjamin–Bona–Mahony equation [4]:

$$u_t + u_x + au^n u_x + u_{xxt} = 0, \quad n \geq 1, \tag{1}$$

where a is a constant and n is the order of nonlinearity involved in the equation. We consider its two special cases which are widely used in nonlinear phenomena. The case $n = 1$, was proposed by Benjamin et al. [4] in 1972. It describes the unidirectional propagation of long waves in certain nonlinear dispersive media. The second case $n = 2$, is the modified Benjamin–Bona–Mahony equation. Due to the importance of the generalized Benjamin–Bona–Mahony equation, a great deal of research work has been carried out to find solitonary, periodic and exact traveling wave solutions of this equation. Several effective techniques including the homogeneous balance method [1] by Abdel Rady et al., an algebraic method [16] by Tang et al., the variable-coefficient balancing-act method [21] by Chen et al., the factorization technique [6] by Estevez

* Corresponding author.
E-mail addresses: mnoor.c@ksu.edu.sa, noormaslam@hotmail.com (M.A. Noor), khakidanoor@hotmail.com (K.I. Noor), waheedasif@hotmail.com, asifwaheed8@gmail.com (A. Waheed), eisasaid@ksu.edu.sa (E.A. Al-Said).

0898-1221/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.camwa.2011.06.060

et al. and the Jacobi elliptic function expansion method [3] by An and Zhang have been investigated for the solitonary periodic and exact traveling wave solutions of this equation. Mammeri [9] found some long time bounds for the periodic Benjamin–Bona–Mahony equation by using a transformation. Gomez et al. [7] have applied tanh–coth method for finding some new periodic and soliton solutions for the generalized BBM and Burgers-BBM equation. Recently, the variational iteration method and the exp-function method have been coupled together by Gomez and Salas [8] to construct traveling wave solutions of this equation.

He and Wu [22] developed the exp-function method and has been used extensively to seek the solitonary, periodic and compacton like solutions of nonlinear differential equations; see [2,5,22,10–15,23–25,17,18] and the references therein. The expression of the exp-function method is more general than the tanh-function method [25]; the solution procedure using Maple, Matlab or Mathematica, is of utter simplicity and the exp-function method is more convenient and effective than other analytic techniques. Noor et al. [11–15,23] have successfully applied the exp-function method for finding soliton, periodic and exact traveling wave solutions of several known partial differential equations like Boussinesq equation, good Boussinesq equation, master partial differential equation, Calogero–Degasperis–Fokas equation, Lax equation and nonlinear evolution equations.

In this paper, we use the exp-function method to construct some new solitonary solutions of the known nonlinear evolution equation such as the Benjamin–Bona–Mahony equation and its variant forms. We present the graphical representation of the soliton solution of Benjamin–Bona–Mahony equations. We hope that this technique can be applied for finding the soliton solutions of other nonlinear evolution equations. It is worth mentioning that the exp-function method has been modified using some novel ideas and techniques; see, for example, [18–20].

2. Exp-function method

We consider the general nonlinear partial differential equation of the type:

$$P\left(u, u_t, u_x, u_t, u_{xx}, u_{xt}, u_{xxt}, \ldots\right) = 0. \tag{2}$$

Using a transformation

$$\eta = kx + \omega t, \tag{3}$$

where k and ω are real constants. We can rewrite Eq. (2) in the following form of nonlinear ordinary differential equation:

$$Q\left(u, u', u'', u''', \ldots\right) = 0, \tag{4}$$

where the prime denotes derivative with respect to η. According to the exp-function method, which was developed by He and Wu [22], we assume that the wave solution can be expressed in the following form:

$$u(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp[n\eta]}{\sum_{m=-p}^{q} b_m \exp[m\eta]} \tag{5}$$

where p, q, c and d are unknown parameters which can be further determined. a_n and b_m are unknown constants. We can rewrite Eq. (5) in the following equivalent form:

$$u(\eta) = \frac{a_c \exp[c\eta] + \cdots + a_{-d} \exp[-d\eta]}{b_p \exp[p\eta] + \cdots + b_{-q} \exp[-q\eta]}. \tag{6}$$

This equivalent formulation plays an important and fundamental role for finding the analytic solutions of problems. To determine the value of c and p, we balance the linear term of highest order of Eq. (4) with the highest order nonlinear term. Similarly, to determine the value of d and q, we balance the linear term of lowest order of Eq. (4) with lowest order nonlinear term.

3. Numerical applications

In this section, we apply the exp-function method to construct some new soliton solutions of the Benjamin–Bona–Mahony and modified Benjamin–Bona–Mahony equations.

Example 3.1 (*[1,3,4,6–9,16]*)**.** Consider the generalized Benjamin–Bona–Mahony equation (1) with $n = 1$, known as Benjamin–Bona–Mahony equation:

$$u_t + u_x + auu_x + u_{xxt} = 0. \tag{7}$$

Introducing a transformation as $\eta = kx + \omega t$, we can covert Eq. (7) into the following ordinary differential equation:

$$(\omega + k)\, u' + a\, k\, uu' + \omega\, k^2 u''' = 0. \tag{8}$$