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Theoretical analysis of large amplitude folding of a single viscous layer

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A B S T R A C T

We present a theoretical model for Large Amplitude Folding (LAF) of a single, viscous layer embedded in a viscous matrix. LAF analysis is rooted in the first order thick-plate analysis but extends it by incorporating two growth rate corrections. 1) Following Fletcher (1974), the growth rate is modified according to the evolution of the wavelength to thickness ratio. 2) A growth rate reduction is introduced based on the rate of arclength shortening, as originally developed by Schmalholz and Podladchikov (2000). Through comparison with numerical models, we show that the simultaneous application of the two corrections in LAF provides a good prediction of the evolution of fold geometry parameters up to large amplitudes irrespective of the particular initial perturbation geometry and viscosity ratio. In the case of the multiple waveforms perturbation, we predict a coupling of the evolution of waveforms. We show that the irregular (non-sinusoidal) or localized final fold shape, commonly observed in nature, can be predicted using LAF.

1. Introduction

The wide range of fold shapes observed in nature can be attributed to the various controlling parameters, e.g., initial layer geometry, amount of shortening, material properties, and folding mechanism. In order to be able to infer these controlling parameters from the final fold shape, their relation to the fold geometry must to be established. Various approaches have been explored: analytical studies (e.g., Biot, 1957, 1961; Fletcher, 1974; Johnson and Fletcher, 1994; Ramberg, 1961; Schmalholz and Podladchikov, 2000), analogue (e.g., Abbassi and Mancktelow, 1992; Cobbold, 1975; Hudleston, 1973b), and numerical modelling (e.g., Dieterich and Carter, 1969; Kocher et al., 2008; Mancktelow, 2001; Parrish, 1973). An up to date review can be found in Hudleston and Treagus (2010).

Folds originate from the growth of small, geometrical irregularities on the layer interfaces (Biot, 1957, 1961; Fletcher, 1974). Any interface perturbation can be represented as a sum of sinusoidal components (e.g., Fletcher and Sherwin, 1978; Mancktelow, 2001). Linear stability analysis predicts that during the small amplitude stages of deformation these components grow independently of each other, each with a specific exponential growth rate (e.g., Abbassi and Mancktelow, 1992; Biot, 1961; Fletcher, 1974). By taking into account the evolution of wavelength and thickness during shortening, the validity of the exponential solution can be extended up to the point when the fold limb dips reach 10–20°. At this stage, the wavelength selection process is considered to lock and the further fold growth takes place at nearly constant arclength and thickness (Fletcher and Sherwin, 1978; Hudleston, 1973a). The correction for the evolution of wavelength and thickness was employed in two- and three-dimensional folding analysis (Fletcher, 1974, 1991; Sherwin and Chapple, 1968). In the case of the two-dimensional analysis, the information about the wavelength selection preserved in the arclength and thickness of large amplitude folds was used to establish a relation between the arclength to thickness ratio, and the layer stretch and the viscosity ratio (Fletcher and Sherwin, 1978).

Another approach for large amplitude folding was presented by Schmalholz and Podladchikov (2000). They attributed the driving force of folding to the rate of arclength shortening rather than the rate of background shortening, as in the previously mentioned approach. This modification results in a slowdown of the amplitude growth, which marks the transition between nucleation and amplification regime of the fold evolution (Schmalholz, 2006b). The amplitude at which the transition occurs is referred to as crossover amplitude (Schmalholz and Podladchikov, 2000). The model was derived based on thin-plate analysis and focused specifically on folds where the initial noise represents the single sinusoidal waveform with dominant wavelength. Based on this model, Schmalholz and Podladchikov (2001) constructed a strain map,

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which allows the viscosity ratio and strain to be inferred. FAS was later extended for three-dimensional folding by Kaus and Schmalholz (2006).

Here, we develop a new model of single layer folding in linear viscous materials that is valid up to large amplitudes. We derive a closed system of ordinary differential equations that describes the evolution of the fold geometry parameters. We base the analysis on the thick-plate solution for the instantaneous growth rate and include both of the above-mentioned corrections, namely 1) the correction for the evolution of the wavelength to thickness ratio and 2) the correction for the slowdown of the amplitude exponential growth. Following Schmalholz and Podladchikov (2000), the latter correction is introduced with correction factor. In our model, which we refer to as LAF, we derive a new expression for the correction factor, which makes the model suitable for the analysis of the evolution of the fold geometry parameters from both single and multiple (random or localized) waveform perturbations. The new expression for the correction factor in case of the multiple waveform perturbation leads to the coupling of the individual waveforms.

To demonstrate the advantages of the simultaneous incorporation of both corrections, we compare our model with the one of Fletcher (1977) and a modified version of Schmalholz and Podladchikov (2000), where the thick- rather than the thin-plate solution is employed (as in Schmalholz, 2006b). In addition, we provide comparisons with finite element models. We show that for single waveform perturbations LAF gives the most accurate results. In the case of multiple waveforms perturbations, LAF is the first model that predicts the fold evolution up to large amplitudes.

The analysis incorporated in LAF advances our understanding of how folds develop up to large amplitudes for both single and multiple waveforms perturbation. Moreover, LAF is characterized by its simplicity and broad applicability, thus provides a new perspective on the study of the folding process.

2. Fold amplitude evolution models (single sinusoidal waveform)

We consider a viscous single layer embedded on both sides in viscous half-spaces. H denotes the thickness of the layer. Both the layer and matrix are linear viscous, homogeneous, isotropic, and incompressible fluids. Perfect welding between the layer and matrix is assumed. In the absence of body forces, the model is subjected to pure shear with a background shortening rate denoted by D_{xx} (where $D_{xx} < 0$). Both layer interfaces are perturbed with a synchronized sinusoidal waveform (further referred to as the waveform)

$$y(x) = A\cos(2\pi x/\lambda) \tag{1}$$

where A is the amplitude, x and y are the spatial Cartesian coordinates, and λ denotes the wavelength of the waveform. The wavenumber k is proportional to the reciprocal of the wavelength

$$k = 2\pi/\lambda \tag{2}$$

Tilde is used to denote normalization of the wavelength and the wavenumber with respect to layer thickness, i.e. $\tilde{\lambda} = \lambda/H$ and $\tilde{k} = kH$.

2.1. Small Amplitude Solution (SAS)

According to the linear stability analysis (Biot, 1957), the amplitude evolution is governed by

$$\frac{\mathrm{d}A}{\mathrm{d}\tau} = A(1+q) \tag{3}$$

where q is the growth rate dependent on the waveform, and $\tau = -D_{xx}t$ is a dimensionless time. Using the thick-plate analysis, Fletcher (1977) obtained an exact result for the growth rates of infinitesimal amplitudes for linear viscous materials

$$q = \frac{4\tilde{k}(1-R)R}{2\tilde{k}(R^2-1) - (R+1)^2\cdot\exp\left(\tilde{k}\right) + (R-1)^2\cdot\exp\left(-\tilde{k}\right)} \tag{4}$$

where R is the viscosity ratio between layer and matrix. The maximum growth rate is experienced by the wavelength of the dominant waveform (further referred to as dominant wavelength) $\tilde{\lambda}^d$ (Biot, 1957).

Due to layer shortening and layer thickening, the normalized wavenumber $\tilde{k}$ of a waveform increases with dimensionless time, which we refer to as waveform evolution. To the first order, the change of $\tilde{k}$ is given by Fletcher (1974).

$$\tilde{k} = \tilde{k}_0\exp(2\tau) \tag{5}$$

The amplitude, to the first order, is found to follow an exponential growth

$$A = A_0\exp\int_0^{\tau}[1+q(\tau')]\mathrm{d}\tau' \tag{6}$$

where τ' denotes the variable of integration. The amplitude evolution can be calculated numerically using e.g., a high-order Runge–Kutta scheme. The maximum amplification is recorded by the wavelength of the preferred waveform (further referred to as preferred wavelength) $\tilde{\lambda}^p$ (Sherwin and Chapple, 1968). Under the approximation that the growth rate spectrum for the logarithm of the wavenumber is symmetric about the dominant wavelength, the preferred wavelength can be estimated by scaling the dominant wavelength with the layer-parallel stretch (Johnson and Pfaff, 1989).

$$\tilde{\lambda}^p = \tilde{\lambda}^d\exp(-\tau) \tag{7}$$

The initial preferred wavelength is $\tilde{\lambda}_0^p = \tilde{\lambda}^d\exp(\tau)$. The model provides a good approximation to the amplitude evolution for limb dips ($\sim A/\lambda$) up to 10–20° (Chapple, 1968; Fletcher and Sherwin, 1978). We refer to this model as the Small Amplitude Solution (SAS).

2.2. Finite Amplitude Solution (FAS)

Schmalholz and Podladchikov (2000) derived the Finite Amplitude Solution (FAS), where perturbed flow in folding is driven by an effective layer shortening rate rather than the applied background shortening rate. An approximation for the effective layer shortening rate is the rate of arclength shortening. We use c to denote the ratio between the rate of change in the fold arclength D_L normalized by the fold arclength L and the rate of background shortening D_{xx}.

$$c = \frac{D_L}{D_{xx}} = \frac{1}{L}\frac{\partial L}{\partial t}\frac{1}{D_{xx}} = -\frac{1}{L}\frac{\partial L}{\partial \tau} \tag{8}$$

The growth rates in FAS are modified according to

$$q_{\mathrm{FAS}} = q\cdot c \tag{9}$$

and we refer to the ratio c as the correction factor. Schmalholz and Podladchikov (2000) derived a formula for the correction factor