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Kinematics of constant arc length folding for different fold shapes

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ABSTRACT

Basic mathematical functions are applied for the two-dimensional geometrical and kinematical analysis of different fold shapes. Relationships between different fold parameters are established and related to the bulk shortening taking place during folding under upper crustal conditions. The bulk shortening taking place during constant arc length folding is mathematically related to the bulk shortening during homogenous pure shear using a particular aspect ratio, which is for folding the ratio of amplitude to half wavelength and for pure shear the ratio of vertical to horizontal length of the deformed, initially square body. The evolution of the fold aspect ratio with bulk shortening is similar for a wide range of fold shapes and indicates that the fold aspect ratio allows a good estimate of the bulk shortening. The change of the geometry of individual layers across a multilayer sequence in disharmonic folding indicates a specific kinematics of multilayer folding, referred to here as “wrap folding”, which does not require significant flexural slip nor flexural flow. The kinematic analysis indicates that there is a critical value for constant arc length folding between shortening values of 30–40% (depending on the fold geometry). For shortening values smaller than the critical value limb rotation and fold amplitude growth are dominating. For shortening larger than this value, faulting, boudinage and foliation development are likely the dominating deformation process during continued shortening. The kinematical analysis of constant arc length folding can be used for estimating the bulk shortening taking place during multilayer folding which is an important component of the deformation of crustal rocks during the early history of shortening. The bulk shortening is estimated for a natural, multilayer detachment fold and the shortening estimates based on the kinematic analysis are compared and supported by numerical finite element simulations of multilayer detachment folding in power-law materials.

1. Introduction

Folding, faulting and layer-parallel homogeneous shortening are three mechanisms for the deformation (shortening) of layered rocks in fold-and-thrust belts (Dixon and Liu, 1992). The research on folds and folding covers a wide range of studies focusing on different topics such as: (1) using and synthesizing mathematical functions to describe fold geometries (Currie et al., 1962; Stabler, 1968; Hudleston, 1973a; De Paor, 1996; Bastida et al., 1999, 2005; Jeng et al., 2002; Aller et al., 2004), (2) analytical solutions employing different rheologies for analyzing folding processes (Chapple, 1968; Johnson and Ellen, 1974; Johnson and Honea, 1975a,b; Biot, 1961, 1964, 1965a,b; Schmalholz et al., 2002), (3) numerical and analogue modeling of single- and multilayer folding investigating dominant wavelengths and amplification rates (Sherwin and Chapple, 1968; Hudleston, 1973a,b; Abbassi and Mancktelow, 1990, 1992; Vacas Peña and Martínez Catalan, 2004; Jeng and Huang, 2008), (4) analyzing the geometry of folded layers using the layer thickness perpendicular to layering and parallel to the fold axial plane as variables (Ramsay, 1967; Hudleston, 1973c; Ramsay and Huber, 1997), (5) investigating the kinematic implications of folding by studying the type and distribution of strain within the folded layers (Johnson and Honea, 1975a, Hudleston et al., 1996; Bastida et al., 2003, 2005, 2007; Bobillo-Ares et al., 2006), and (6) studying folds in relation to other structures such as faults, boudins, foliations and lineations (Sengupta, 1983; Mawer and Williams, 1991; Kobberger and Zulauf, 1995; Kraus and Williams, 1998; Mitra, 2003; Savage and Cook, 2003).

In this study, we apply kinematic models of constant arc length folding for estimating the bulk shortening taking place during

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folding. The kinematic models are based on geometrical models describing observed fold shapes in profile view. Fold profiles are sections (orthogonal to the fold axis) of folded lines and their geometry can be approximated with mathematical functions. Different functions have been suggested for this purpose, which can be grouped in two major categories: non-periodic functions (Hudleston, 1973a; De Paor, 1996; Bastida et al., 1999, 2005; Aller et al., 2004; Bastida et al., 2005) and periodic functions (Currie et al., 1962; Stabler, 1968; Hudleston, 1973a; Bastida et al., 1999; Jeng et al., 2002).

This study briefly summarizes and builds on previous work on the geometry of a single folded layer in a two-dimensional profile (e.g. Stabler, 1968; Hudleston, 1973a,b; Bastida et al., 1999). The study applies basic mathematical procedures for shortening analysis of folds, and the quantities limb dip, interlimb angle, arc length, curvature, aspect ratio (i.e. ratio of fold amplitude to half wavelength) and area under the folded layer are analyzed for different fold types. The presented kinematical analysis is applied to estimate the bulk shortening that took place during folding of a natural multilayer detachment fold. The results of the kinematical analysis are compared with an analytical solution for the mechanical process of viscous single-layer folding and with numerical finite element simulations of ductile, multilayer detachment folding. The comparisons show that the kinematical folding analysis can provide good approximations for the bulk shortening during folding. Potential applications of the presented analyses for estimating the shortening, the variations of the geometry in a folded sequence, and the fold growth are discussed.

2. Fold geometry

2.1. Representing fold geometries with mathematical functions

Fitting all fold shapes with one type of mathematical function is not suitable because geometries of natural folds vary significantly. For example, methods for fitting folds with ellipses (Mertie, 1959) are unsuitable for an accurate representation of fold shapes and many common fold styles (e.g. chevron folds) cannot be represented at all.

Representing fold shapes with Fourier series received most attention (see Norris, 1963; Chapple, 1964, 1968; Harbaugh and Preston, 1965, Stabler, 1968; Hudleston, 1973a; Ramsay and Huber, 1997) because many folds are naturally periodic. The Fourier analysis of fold shapes is useful for sinusoidal fold shapes (see Stabler, 1968; Hudleston, 1973a), however, it has some drawbacks when applied to other fold shapes (see Bastida et al., 2005).

Several studies (Bastida et al., 1999; Aller et al., 2004; Bastida et al., 2005; Lisle et al., 2006) suggested a range of functions for representing fold shapes. Bastida et al. (1999) suggested a power function:

$$\frac{y}{y_0} = \left(\frac{x}{x_0}\right)^n \tag{1}$$

in which n characterizes the fold shape, x_0 and y_0 are the coordinates of the inflexion point on the fold limb, and y and x are the vertical (i.e. parallel to the fold axial plane) and horizontal coordinates, respectively. In order to have a common coordinate system[2] and to analyze the fold limb between an inflection point at the origin of the coordinate system and the fold hinge we use a similar function:

$$y = 4\frac{A}{w}(1-(1-x)^n), \tag{2}$$

where A and w are the amplitude and wavelength of the fold, respectively. This equation does not have the inconveniences of Eq. (1) which are described in Bastida et al. (2005). Equation (2) can be modified to a function for the variable p which is the aspect ratio of the fold and defined as the ratio of fold amplitude to half the fold wavelength (Twiss, 1988):

$$y = 2p(1-(1-x)^n). \tag{3}$$

For $n = 1$ Eq. (3) is

$$y = 2px, \tag{4}$$

and represents ideal chevron folds.

For $n = 2$ Eq. (3) is

$$y = 2p\left(2x - x^2\right), \tag{5}$$

and represents parabolic folds.

Applying a power of 0.5 to the right term in braces in Eq. (5) results in

$$y = 2p\left(2x - x^2\right)^{0.5}, \tag{6}$$

and represents ellipsoidal folds.

Using $n > 2$ in Eq. (3) produces double hinge fold shapes (see Table 1).

Equations (3)–(6) and the Fourier series for the first harmonic can be used to describe a wide variety of fold shapes. These equations are not based on one type of function and, therefore, lack the continuity of fold shapes which is for example a feature of the power function of Eq. (1) (Bastida et al., 1999). However, because of the wide diversity of natural fold shapes we represent here basic fold shapes with specific functions that best fit the observed fold shape (see Table 1).

Cuspate folds are not represented with any specific function because all functions listed in Table 1, except the linear function, represent different shapes of cuspate folds when they are mirrored against the chord of the fold's quarter wavelength.

2.2. Basic geometrical implications of fold shapes

The interlimb angle, i, of upright symmetrical chevron folds is related to their aspect ratio, p:

$$i = 2\arctan\frac{1}{2p}. \tag{7}$$

Ghent and Hansen (1999) used an equation similar to Eq. (7), however, what they termed fold wavelength is in fact the fold's half wavelength. The interlimb angle is increasingly smaller for sinusoidal, parabolic and double hinge folds for the same value of p. The maximum dip of the fold limb, l, at the inflection point of upright

Table 1
General functions for different fold geometries used in this study for analyzing fold geometry and kinematics.

Fold type	General function
Chevron	$y = 2px$
Sinusoidal	$y = 2p\sin(\pi/2x)$
Parabolic	$y = 2p(2x - x^2)$
Ellipsoidal	$y = 2p(2x - x^2)^{0.5}$
Double hinge (box)	$y = 2p(1-(1-x)^n)$; $n > 2$

[2] Following Hudleston (1973a) and Ramsay and Huber (1997), this paper assumes the y axis of the coordinate system passing through inflection point of the fold and parallel to the axial surface of the fold. The x axis also passes through the inflection point, and is perpendicular to the y axis.