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Clustering of fracture orientations using a mixed Bingham distribution and its application to paleostress analysis from dike or vein orientations

Atsushi Yamaji*, Katsushi Sato

Division of Earth and Planetary Sciences, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan

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ABSTRACT

The clustering and classification of fracture orientations are important in rock mechanics and in brittle tectonics, the latter of which includes the paleostress analysis of extension fractures hosting dikes or mineral veins. Here, we present an unsupervised clustering method for the orientations of extension fractures using mixed Bingham distributions. The method not only detects the elliptical clusters and girdles made by the poles to such planar features, but also determines the appropriate number of those groups by means of Bayesian information criterion (BIC) without a priori information. The method was tested with artificial data sets, and successfully detected the assumed groups, when the clusters had little overlaps. However, clusters with the common maximum concentration orientation and large aspect ratios were distinguished, provided that their minimum concentration orientations were separated by a large angle. Our method separated two stress states from natural data from a Miocene dike swarm in SW Japan. The method also evaluated the probabilities of the stresses to form each of the dike.

1. Introduction

The clustering of orientation data is important in various branches of science and engineering. Discontinuity orientations in rock material are carefully observed when surface and underground excavations are made from efficiency and safety standpoints (Priest, 1993). Their orientation distribution is important for wellbore stability (Chen et al., 2008) and groundwater hydrology (e.g., Panda and Kultilake, 1999; Ohtsu et al., 2008). Accordingly, various clustering techniques for the orientations have been proposed since the 1970s by researchers mainly in civil engineering (e.g., Shanley and Mahtab, 1976; Wallbrecher, 1978; Hammah and Curran, 1998, 1999; Peel et al., 2001; Marcotte and Henry, 2002; Klose et al., 2005; Jimenez-Rodriguez and Sitar, 2006). Dortet-Bernadet and Wicker (2008) suggest that Peel et al. (2001), who clustered rock joints, stimulated researchers in other fields of science to tackle the problem.

Such clustering is important for understanding brittle tectonics as well. The orientations of healed microcracks (Lespinasse and Pécher, 1986; Kowallis et al., 1987) and joints (Whitaker and Engelder, 2005) are thought to indicate paleostress orientations. In addition, dike and vein orientations are used to infer all the axes of the paleostress at the time of the vein or dike formation (Baer et al., 1994; Jolly and Sanderson, 1997; Yamaji et al., 2010). The clustering of vein orientations was used by Ahmadhadi et al. (2008) to infer the timing of folding. The clustering of fracture orientations has potential for investigating polyphase tectonics.

Fault-slip analysis has been used to study polyphase tectonics (e.g., Etchecopar et al., 1981; Nemcok and Lisle, 1995; Yamaji, 2000; Shan et al., 2003; Sato, 2006; Yamaji et al., 2006). The fault-slip data resulting from such tectonics are called heterogeneous. Likewise, we call a data set heterogeneous, if the data are collected from the fractures that should be classified into some groups with different origins.

In this paper we present a clustering method for dealing with heterogeneous orientation data. It is assumed that the poles to planar features of the same origin make an elliptical cluster or a girdle that is approximated by a Bingham distribution (Bingham, 1974). This is the simplest orientation distribution to delineate them (Fig. 1), and is easily related with the dilation of fractures by overpressured fluids (Baer et al., 1994; Jolly and Sanderson, 1997; Yamaji et al., 2010). Our method simultaneously fits a few Bingham distributions to a set of heterogeneous data. That is, a mixed Bingham distribution is fitted to them. Our numerical technique not only detects and separates the clusters and girdles, but also determines their number from the orientation data themselves.

* Corresponding author. Fax: +81 75 753 4189.
E-mail address: yamaji@kueps.kyoto-u.ac.jp (A. Yamaji).

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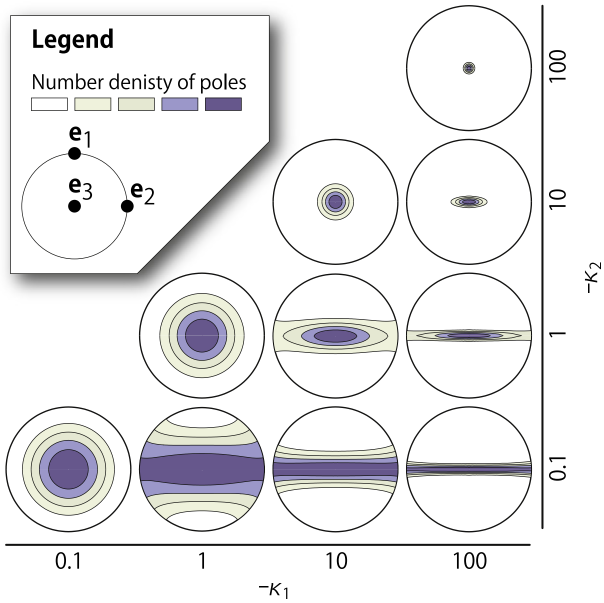

Fig. 1. Equal-area projections showing the probability densities of the Bingham distributions with different κ_1 and κ_2 values, both of which are negative in sign. The distributions have orthorhombic symmetry, meaning that they are symmetric with respect to the planes perpendicular to the unit vectors, $\mathbf{e}_1$, $\mathbf{e}_2$ or $\mathbf{e}_3$. Note that the stereograms have different contour intervals: the range between the minimum and maximum densities, i.e., $\mathcal{P}_B(\mathbf{e}_1|\mathbf{K},\mathbf{E})$ and $\mathcal{P}_B(\mathbf{e}_3|\mathbf{K},\mathbf{E})$, is divided into 5 intervals.

The method was tested with artificial data sets to demonstrate its resolution, and with natural data sets from a dike swarm.

The analysis of clustering of dike orientations and vein orientations will stimulate structural geologists and researchers in related areas. Once fractures are classified, radiometric dating, paleomagnetic, petrological and geochemical analyses, etc., of the members of each class shed new light on the formation of the dike and vein clusters and on their tectonic, volcanological and hydrological implications.

2. Bingham and mixed Bingham distributions

The Bingham distribution is the simplest extension of the multivariate normal distribution to the three-dimensional orientation distribution of lines (e.g., Love, 2007). It is convenient to consider antipodally distributed points on a sphere to represent the lines that meet at the center of the sphere. The Bingham distribution is depicted by a girdle or an elliptical cluster of such points.

An elliptical cluster or a girdle has orthorhombic symmetry if it is described by the Bingham distribution. That is, it has the three symmetry axes that meet at right angles; two of them indicate the orientations of maximum and minimum concentrations. The remaining axis is known as the orientation of intermediate concentration. Following Love (2007), we use the unit column vectors, $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$, to refer to the orientations of the minimum, intermediate and maximum concentrations, respectively (Table 1). The cluster center is represented by $\mathbf{e}_3$, which is identified with the σ_3-axis in Section 5 (Baer et al., 1994; Jolly and Sanderson, 1997; Yamaji et al., 2010).

The paired parameters, κ_1 and κ_2, distinguish uniform, elliptical and girdle distributions (Fig. 1): They are negative in sign, and their absolute values, $|\kappa_1|$ and $|\kappa_2|$, indicate the concentration of data points from $\mathbf{e}_3$ to $\mathbf{e}_1$ and from $\mathbf{e}_1$ to $\mathbf{e}_2$, respectively, on the sphere. A uniform distribution is indicated by $\kappa_1 = \kappa_2 = 0$. Circular and elliptical distributions are indicated by $\kappa_1 = \kappa_2 < 0$ and $\kappa_1 < \kappa_2 < 0$,

Table 1
List of symbols. Superscript at the upper left and upper right of a symbol denote, respectively, the number of iterations in the EM algorithm and the consecutive number of Bingham components in a mixed Bingham distribution. Circumflex accents indicate the quantities of the mixed Bingham distribution optimized for a data set.

BIC	Bayesian information criterion
$\mathbf{E}$	Orthogonal matrix representing the symmetry axes of a Bingham distribution
$\mathbf{e}_1$	The minimum concentration axis of a Bingham distribution
$\mathbf{e}_2$	The intermediate concentration axis of a Bingham distribution
$\mathbf{e}_3$	The maximum concentration axis of a Bingham distribution
K	The number of Bingham component of a mixed Bingham distribution
$\mathbf{K}$	Diagonal matrix with the diagonal components, κ_1, κ_2 and 0
$\mathscr{L}$	Logarithmic likelihood function
N	The number of data
$\mathcal{P}_B$	Probability density function of Bingham distribution
$\mathcal{P}_{mB}$	Probability density function of mixed Bingham distribution
$\mathbf{v}$	Unit vector normal to a fracture plane
$\mathbf{v}_n$	$\mathbf{v}$ of the nth fracture
$\mathbf{x}$	A five-dimensional vector representing a Bingham distribution
z_n^k	The membership of the nth datum to the kth Bingham component or The responsibility of the kth one for the nth datum
$\boldsymbol{\theta}$	The set of the K vectors representing Bingham distributions
κ_1, κ_2	Concentration parameters of a Bingham distribution ($\kappa_1 \le \kappa_2 \le 0$)
ϖ	The set of K mixing coefficients
ϖ^k	The mixing coefficient of the kth Bingham component
σ_1, σ_2, σ_3	Principal stresses ($\sigma_1 \ge \sigma_2 \ge \sigma_3$)
Φ	Stress ratio

respectively. Girdle distributions are denoted by the parameters that satisfy $\kappa_1 \ll \kappa_2 \lessapprox 0$.

If points on a unit sphere obey Bingham distribution, they have the probability density (Love, 2007)

$$\mathcal{P}_B(\mathbf{v}|\mathbf{K},\mathbf{E}) = \frac{1}{A}\exp\left(\mathbf{v}^T\mathbf{E}^T\mathbf{K}\mathbf{E}\mathbf{v}\right),$$

where $\mathbf{v}$ is the unit vector representing an orientation, A is the normalization constant, T indicates matrix transpose, $\mathbf{E} = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is the orthogonal matrix representing the attitude of the Bingham distribution, and $\mathbf{K} = \mathrm{diag}(\kappa_1, \kappa_2, 0)$. The distribution has five degrees of freedom: three for the orthonormal vectors, $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$, and two for the concentration parameters. Accordingly, the parameters of the distribution are represented by a position vector, $\mathbf{x}$, in a five-dimensional parameter space (Appendix A). That is, the paired parameters, $\{\mathbf{K},\mathbf{E}\}$, have a one-to-one correspondence with a point in the space. We refer $\mathcal{P}_B(\mathbf{v}|\mathbf{x})$ to the probability density of the Bingham distribution with the parameters that are denoted by $\mathbf{x}$.

The Bingham distribution is so flexible as to denote either an elliptical cluster or a girdle made by the poles to fractures. Accordingly, it is useful to assume that a heterogeneous set of orientation data obeys the mixed Bingham distribution, which has the probability density

$$\mathcal{P}_{mB}(\mathbf{v}|\boldsymbol{\theta},\boldsymbol{\varpi}) = \sum_{k=1}^{K}\varpi^k\mathcal{P}_B\left(\mathbf{v}\middle|\mathbf{x}^k\right), \quad (1)$$

where K is the number of elliptical clusters or girdles, ϖ^k is the compounding ratio or the mixing coefficient (Bishop, 2006) of the kth Bingham distribution of which parameters are represented by $\mathbf{x}^k$. The coefficients satisfy $0 < \varpi^k \le 1$ and $\varpi^1 + \cdots + \varpi^K = 1$: ϖ^k means the significance of the kth subset. The argument, $\boldsymbol{\theta}$, of the function $\mathcal{P}_{mB}$ in Eq. (1) stands for all the K vectors:

$$\boldsymbol{\theta} = \left\{\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^K\right\}, \quad (2)$$

and another argument of the function is $\varpi = \{\varpi^1, \varpi^2, \ldots, \varpi^K\}$. Fig. 2 shows an example with the parameters, $K = 2$, $\varpi^1 = 0.4$ and