Journal of Structural Geology 33 (2011) 1178–1192

Contents lists available at ScienceDirect

Journal of Structural Geology

journal homepage: www.elsevier.com/locate/jsg

Effect of finite strain on clast-based vorticity gauges

Donald W. Stahr III *, Richard D. Law

Department of Geosciences, Virginia Tech, Blacksburg, VA 24061, USA

ARTICLE INFO

Article history:
Received 28 September 2010
Received in revised form
29 March 2011
Accepted 8 May 2011
Available online 13 May 2011

Keywords:
Ductile flow
Mylonite
Numerical model
Rigid clasts
Strain
Vorticity analysis

ABSTRACT

Clast-based vorticity gauges utilize orientations of grains assumed to have behaved as isolated rigid particles suspended in a flowing viscous matrix. A fundamental assumption behind use of the method is that sufficient strain has accumulated for high aspect ratio grains to rotate into positions approaching their stable sink orientation, and that clasts below a critical aspect ratio may be observed in any orientation relative to the flow plane. We constructed a numerical model to explore the effect of variable finite strain on development of the orientation distribution of a large population of rigid clasts embedded in a viscous medium for end-member pure and simple shear and for several distinct general shear flows. Our model predicts the technique will tend to produce vorticity overestimates for lower vorticity flows for a wide range of finite strain. The model also indicates that clast populations in moderate to high vortical flows tend to develop shape preferred orientations that closely resemble those expected for flows of lower vorticity. We conclude that clast-based methods are not effective for extracting detailed kinematic information from a mylonite deformed in a flow with arbitrary boundary conditions. In fact, it appears that most general shear flows continued long enough to develop moderate–high finite strains will tend to produce a clast orientation distribution that will yield a visual estimate of the critical aspect ratio that suggests approximately equal contributions of pure and simple shear components.

1. Introduction

Since the introduction of kinematic vorticity into the geological literature (McKenzie, 1979; Means et al., 1980), and the development of methods for extracting these data from naturally deformed rocks (e.g., Passchier, 1986, 1987; Vissers, 1989; Wallis, 1992; Wallis et al., 1993; Simpson and De Paor, 1993), structural studies of orogenic belts have increasingly focused on determining the boundary conditions of flow during ductile deformation in high-strain zones. Results of these investigations have repeatedly shown crustal-scale shear zones from a wide array of tectonic settings involved a departure from ideal simple shear (e.g., Passchier, 1987; Vissers, 1989; Wallis et al., 1993; Xypolias and Doutsos, 2000; Law et al., 2004; Jessup et al., 2006, 2007; Bailey et al., 2007; Johnson et al., 2009). The implications of these results are significant for several reasons. Consider a shallowly dipping mylonite zone, a common feature in orogenic hinterlands, deforming by simultaneous pure and simple shearing [here we follow previous authors (e.g., Ramberg, 1975) in using the *suffix-ing* to emphasize terms related to the deformation process]. The pure shearing component of an isochoric, plane strain, sub-simple shearing deformation causes thinning perpendicular to the zone boundaries. Strain compatibility arguments require that material must simultaneously stretch parallel to the shear zone boundary. Material in such a narrowing-lengthening shear zone (Simpson and De Paor, 1993; Tikoff and Fossen, 1999) is likely directed toward the synorogenic topographic surface, causing a material flux from lower to higher crustal levels (i.e., from orogenic core to foreland). Purely geometric arguments indicate the magnitude and rate of extrusion of material increase rapidly from the core to foreland of the orogen forcing an increase in strain rate at higher structural levels (Law, 2010). Such coupling of middle and shallow crustal levels may help drive deformation in the orogenic foreland. This simple example illustrates that reliable methods of determining kinematic parameters from high-strain zones are critically important for meaningful interpretation of structural evolution within such ductile deformation zones.

Several vorticity gauges, including: (1) deformed vein sets (Talbot, 1970; Hutton, 1970; Passchier, 1986); (2) clast-based gauges (Passchier, 1987; Simpson and De Paor, 1993; Wallis et al., 1993); (3) quartz petrofabric and strain ratio (R_{XZ}/β) (Wallis, 1992, 1995); (4) oblique dynamically recrystallized grain shape foliation (Wallis, 1995); R_{XZ}/δ method of (Xypolias (2009, 2010); (5) angle between macroscopic foliation and shear zone boundary (R_{XZ}/θ) (Tikoff and Fossen, 1995); and (6) flanking structures (Grasemann and Stüwe, 2001) have been applied to natural rocks. Clast-based vorticity

* Corresponding author. Tel.: +1 5402319740; fax: +1 5402313386.
E-mail address: dstahr@vt.edu (D.W. Stahr).

0191-8141/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsg.2011.05.002

gauges are the most commonly applied to natural samples due to: (1) their relative simplicity and rapid application; and (2) many of the assumptions required for the technique to be valid are apparently met (see Passchier, 1987). Early theoretical models published by Masuda et al. (1995) indicated that clast-based techniques may be useful to broadly discriminate between coaxial and non-coaxial flow. However, even with the increased use during the last 15 years (e.g., Passchier, 1987; Vissers, 1989; Wallis et al., 1993; Simpson and De Paor, 1997; Holcombe and Little, 2001; Xypolias and Koukouvelas, 2001; Bailey and Eyster, 2003; Law et al., 2004; Carosi et al., 2006; Jessup et al., 2006, 2007; Xypolias and Kokkalas, 2006; Bailey et al., 2007; Marques et al., 2007; Thigpen et al., 2010), no advance has been made on understanding the role finite strain plays on the evolution of clast orientation distributions for different flow types.

Finite strain magnitude is critically important in all vorticity estimation methods as it is either an explicit parameter (e.g., R_{XZ}/θ, R_{XZ}/β, and R_{XZ}/δ methods), or for the clast-based method in particular, it is tacitly assumed that sufficient strain has accumulated for high aspect ratio grains to have rotated into their stable positions. Because finite strain is a fundamental parameter for determining the porphyroclast orientation distribution produced during deformation, we view the lack of knowledge of strain state as a limit on the usefulness of vorticity estimates made solely from clast-based techniques, and argue that multiple techniques should be used to constrain deformation kinematics.

In this paper we first review the mathematical theory necessary to describe pure, simple, and sub-simple shearing flow and use this theoretical framework to model a large population of rigid elliptical objects in viscous flows of variable kinematic vorticity and at a wide range of finite strains. Our primary interest lies in discovering if there exists a single value of finite strain necessary to produce a well-organized orientation distribution for different flow types. To this end we applied the governing equations (and therefore assumptions and limitations) derived in the seminal paper by Ghosh and Ramberg (1976). Some surprising behavior is predicted at moderate to high kinematic vorticity and high finite strains. Implications of these results are discussed in a geological context.

2. Mathematical framework

2.1. Description of flow and progressive deformation

The velocity field about a point in a deforming continuum is described by the velocity gradient tensor, **L**,

$$\mathbf{v} = \mathbf{L}\mathbf{x} \tag{1}$$

where **v** is a velocity vector, or the time derivative of position vector **x** (i.e., $\mathbf{v} = \mathrm{d}\mathbf{x}/\mathrm{d}t$). The associated velocity gradient equations become (Means et al., 1980):

$$v_i = L_{ij}x_j \tag{2}$$

where v_i are the velocity components at position x_j at an instant in time, and

$$L_{ij} = \frac{\partial v_i}{\partial x_j} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$$

(see Malvern, 1969, p. 146) are the spatial velocity gradients for a two-dimensional flow (see Fig. 1). If the velocity gradient tensor components L_{ij} are constant the flow is homogeneous (Means et al., 1980). For isochoric plane strain monoclinic flow with simultaneous pure and simple shearing, **L** may be written as

$$\mathbf{L} = \begin{bmatrix} \dot{\varepsilon}_x & \dot{\gamma} \\ 0 & \dot{\varepsilon}_y \end{bmatrix} \tag{3}$$

where $\dot{\varepsilon}_x$ is the pure shearing strain rate and $\dot{\gamma}$ is the simple shearing strain rate, here taken perpendicular and parallel to the abscissa, respectively. Setting $\dot{\varepsilon}_y = -\dot{\varepsilon}_x$ forces the deforming material to be incompressible. The eigenvectors, ξ_i, of **L** give the orientations of the flow apophyses (Ramberg, 1975; Passchier, 1988).

The velocity gradient tensor, **L**, may be decomposed into the symmetric stretching tensor, $\dot{\mathbf{S}}$, and skew-symmetric vorticity tensor, **W** (Malvern, 1969, p. 147; Bobyarchick, 1986)

$$\mathbf{L} = \dot{\mathbf{S}} + \mathbf{W} \tag{4}$$

where

$$\dot{\mathbf{S}} = \begin{bmatrix} \dot{\varepsilon}_x & \frac{1}{2}\dot{\gamma} \\ \frac{1}{2}\dot{\gamma} & \dot{\varepsilon}_y \end{bmatrix} \tag{5}$$

and

$$\mathbf{W} = \begin{bmatrix} 0 & \frac{1}{2}\dot{\gamma} \\ -\frac{1}{2}\dot{\gamma} & 0 \end{bmatrix}. \tag{6}$$

The eigenvectors and eigenvalues of $\dot{\mathbf{S}}$ provide information on the orientation and magnitude of the instantaneous stretching axes (ISA$_i$) and instantaneous stretching rates ($\dot{s}_i$) of the flow, respectively. The vorticity tensor, **W**, has components of angular velocity and describes the rotation rate of elements in the deforming material.

The kinematic vorticity number, W_k, is a useful way of quantifying the instantaneous non-coaxiality of the flow at a point in space and an instant in time, and has a unique value for any distinct flow. By definition, W_k is an instantaneous quantity, but for the steady flows considered here the vorticity number remains constant during progressive deformation. The quantity s_r, defined as the ratio of pure to simple shearing strain rate, $s_\mathrm{r} = \dot{\varepsilon}_x/\dot{\gamma}$ (Ghosh and Ramberg, 1976), is also a measure of the degree of non-coaxiality of the flow and may be expressed as a function of the kinematic vorticity number by the relation (Ghosh, 1987, Eq. (9))

$$s_\mathrm{r} = \frac{1}{2}\sqrt{\frac{1}{W_k^2} - 1}. \tag{7}$$

Conversely, the kinematic vorticity may be calculated from knowledge of the instantaneous pure and simple shearing strain rates by the relation

$$W_k = \cos\left[\tan^{-1}\left(2\frac{\dot{\varepsilon}_x}{\dot{\gamma}}\right)\right] \tag{8}$$

or more simply

$$W_k = \cos(\alpha) \tag{9}$$

where α is the acute angle between the eigenvectors (ξ_i) of **L** (for derivation see Bobyarchick, 1986). By choosing appropriate values for s_r or W_k we can form a velocity gradient tensor, **L**, to produce the velocity field of a deformation of any vorticity number of interest.

From (8) it is clear that identical W_k values result from any combination of $\dot{\varepsilon}_x$ and $\dot{\gamma}$ that yield the same s_r value. Thus, any choice of $\dot{\varepsilon}_x$ and $\dot{\gamma}$ that yield the same ratio give rise to identical velocity fields; only the time required to accumulate a finite strain state will vary. Note from (7) that s_r increases without bound as