Journal of Structural Geology 30 (2008) 1013–1023

Contents lists available at ScienceDirect

Journal of Structural Geology

journal homepage: www.elsevier.com/locate/jsg

Numerical modelling of the effect of matrix anisotropy orientation on single layer fold development

Thomas Kocher [1], Neil S. Mancktelow *, Stefan M. Schmalholz

Geologisches Institut, ETH Zürich, Sonneggstrasse 5, CH-8092 Zürich, Switzerland

ARTICLE INFO

Article history:
Received 22 November 2007
Received in revised form 7 April 2008
Accepted 15 April 2008
Available online 29 April 2008

Keywords:
Planar anisotropy
Single layer folds
Asymmetric folds
Numeric modelling
Finite element modelling
Viscous flow

ABSTRACT

The influence of matrix anisotropy of variable orientation on single layer folding is investigated using finite element models. Both linear (Newtonian) and power-law viscous materials are considered. The results show that the available isotropic analytical solution, when modified to include an appropriate approximation for the anisotropic viscosity, accurately predicts growth rates at small amplitude for planar anisotropy oriented at $\alpha = 45^\circ$ to the competent layer for a wide range of normal viscosity ratios between single layer and matrix ($\mu_c = 10, 100$) and degrees of anisotropy ($\delta =$ normal viscosity/shear viscosity $= 2, 12, 25$). For high normal viscosity ratio ($\mu_c = 100$), the deviation from the analytical solution for other orientations increases with increasing degree of anisotropy but still remains relatively small ($<5\%$ for $\delta = 25$). For low normal viscosity ratio ($\mu_c = 10$), the differences for high δ are more significant and for $\alpha \neq 0^\circ, 45^\circ$, or 90° also depend on the imposed boundary conditions. However, if carefully applied, the analytical solution does provide a benchmark test for numerical codes that include oblique anisotropy. The numerical models at both small and finite amplitude show that a tight control on the boundary conditions is crucial for experiments with anisotropic materials, especially when the anisotropy is oblique to the boundaries. Analogue experiments with anisotropic materials, where boundary conditions are more difficult to control, must therefore be designed and interpreted with caution. Matrix anisotropy initially oriented obliquely with regard to the maximum shortening direction results in asymmetric buckle folds in the single layer and asymmetric chevron folds in the matrix, even if the deformation is purely coaxial. This is true for both linear and power-law materials and for a range of boundary conditions, both free and constrained. Asymmetric natural fold structures in anisotropic material do not therefore necessarily imply a component of non-coaxial flow.

1. Introduction

As shown in an earlier study (Kocher et al., 2006), layer-parallel anisotropy in the matrix has a strong influence on the infinitesimal and large amplitude stages of single layer folding. The occurrence of an internal instability in the matrix (e.g. Biot, 1965; Cobbold et al., 1971; Cobbold, 1976; Latham 1985a,b; Fletcher, 2005), and its interference with the single layer of higher viscosity, cause substantial changes in growth rates, dominant wavelengths, amplification history, and finite structure pattern compared to an isotropic material. In this previous study, we specifically considered the situation where a pre-existing planar anisotropy is fixed to material points from the onset of deformation and initially parallel to an embedded, more competent single layer. Natural examples would be finely layered sedimentary rocks, such as turbidites or radiolarites, or metamorphic rocks with a strong foliation or banding developed due to metamorphic segregation. In these cases, the bulk anisotropic behaviour reflects the stacking of layers with different viscosities or a set of closely-spaced slip surfaces (corresponding to the "IMSS fluid" of Fletcher, 2005; see also Cobbold et al., 1971). The anisotropy therefore remains parallel to the layer boundaries during subsequent deformation if the material distribution is not altered by metamorphic or metasomatic processes. Because multilayered rocks are common in nature, this is an important case to consider in detail. However, it is only one end-member of a more general situation, where both the degree of the anisotropy and its orientation relative to the layer may vary both initially and during deformation. For example, the approximately planar foliation in natural slates and schists is typically oblique to bedding or layering. In polydeformed terrains, subsequent deformation leads to a crenulation or kinking of the foliation and to second-phase folding of the layering. In this case, the obliquity of the foliation may be expected to influence the dynamics of buckling and the geometry of folds developed in the layering.

* Corresponding author.
E-mail address: neil.mancktelow@erdw.ethz.ch (N.S. Mancktelow).

[1] Present address: Shell International Exploration and Production B.V., Kesslerpark 1, 2288 GS, Rijswijk, The Netherlands.

0191-8141/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsg.2008.04.006

In this paper, finite element models are used to consider pure shear deformation of an isotropic competent layer embedded in a rock matrix with differing initial degrees of planar anisotropy and differing initial orientations of this planar anisotropy relative to the layer. Both Newtonian and non-linear (power-law) viscous materials are considered. In particular, it is investigated: (1) if the analytical theories of Fletcher (1974) and Smith (1977) can still be applied to determine growth rates and dominant wavelengths; (2) if oblique anisotropy might cause asymmetric folds to develop in a background flow field of pure shear; and (3) the influence of oblique anisotropy on the matrix deformation processes.

2. Numerical method

The numerical experiments were performed using the finite element code FLASH (Kocher, 2006) and an equivalent code of Mancktelow (e.g. Viola and Mancktelow, 2005; Passchier et al., 2005), which solve the Stokes equations in combination with a non-linear anisotropic power-law rheology in the absence of gravity. Nine-node quadratic elements for velocity discretization are combined with a linear pressure approximation (three degrees of freedom) to avoid chessboard patterns in pressure fields (e.g. Cuvelier et al., 1986; Poliakov and Podladchikov, 1992). The implementation of anisotropic viscosity in these codes follows Mühlhaus et al. (2002a,b; their Eq. (8) in both publications), as outlined in Kocher (2006) and Kocher et al. (2006, Appendix A). To verify the results, both codes were tested on a number of model setups for which analytical solutions are available, including the analytical solutions of Fletcher (1974) for folding in linear and non-linear viscous material, and Schmid and Podladchikov (2003) for the stress and strain rate fields around an elliptical inclusion in Newtonian fluids.

3. Influence of anisotropy orientation on growth rate spectra of single layer folds

The analytical solutions of Fletcher (1974) and Smith (1975, 1977) predict the growth rates and dominant wavelengths of single layer folds in power-law viscous material at infinitesimal amplitudes. These two independent analytical derivations give equivalent results for the growth rate spectra and here the equation of Fletcher (1974) is used. In Kocher et al. (2006), it was demonstrated that this analytical solution also allows the determination of growth rates for layers embedded in a linear viscous anisotropic matrix. This solution makes use of a proposal by Biot (1965) that the bulk viscosity of an infinite anisotropic half-space can be approximated by $\mu = \sqrt{\mu_n \mu_s}$, where μ_n and μ_s are the normal and shear viscosity of the material. The resulting analytical growth rate was shown to be in good, though not perfect, agreement with the results obtained from finite element analysis (max. error $\leq 5\%$ for the chosen parameters, cf. Fig. 2d of Kocher et al., 2006).

However, it has not yet been established if the analytical solution for growth rates in anisotropic material also applies to the more general case of a non-layer parallel anisotropy orientation. To check this, growth rate spectra of a single isotropic Newtonian layer embedded in an anisotropic Newtonian matrix were calculated for a normal viscosity contrast of $\mu_c = 10$ and 100. For each of three degrees of anisotropy ($\delta = \mu_n/\mu_s = 2, 12$, and 25), growth rate values were calculated for an angle between the competent layer and the plane of anisotropy of $\alpha = 0°$, 20°, 45°, 70° and 90°.

The resulting growth rate spectra are shown in Fig. 1. The following observations can be made from these plots. (1) The numerical results are symmetrical about an anisotropy inclination of 45° to the competent layer. For example, the growth rates for an anisotropy oriented at 0° and 90° or at 20° and 70° to the layer are identical because these orientations are equally inclined relative to the 45° orientation. This is to be expected because the constitutive (or material) operator relating stress to strain rate in the anisotropic matrix is symmetric (Mühlhaus et al., 2002a,b; Kocher et al., 2006, Appendix A) and therefore insensitive to a switch in axes (equivalent to a reflection across the 45° orientation). (2) The analytical solution of Fletcher (1974) is best approximated by the numerical results if the anisotropy is oriented at 45° to the competent layer. (3) The numerical results for $\alpha = 0°$ or 90° – for which the analytical solution was initially proposed – show a good fit to the predicted analytical values for high viscosity contrast (e.g. $\mu_c = 100$ in Fig. 1a–c). (4) Overall, the model growth rates increasingly deviate from the analytical curve with increasing degree of anisotropy but, for high viscosity contrast ($\mu_c = 100$), the maximum deviation still remains relatively small (<5% for $\delta = 25$ and $\alpha = 20°$ or 70° in Fig. 1c). (5) For $\alpha \neq 45°$, the fit of the numerical results to the analytical solution deteriorates with decreasing normal viscosity contrast between matrix and layer (e.g. $\mu_c = 10$ in Fig. 1d–f). (6) The growth rates for $\alpha = 0°$ or 90° are generally lower than the theoretical curve, whereas those for $\alpha = 20°$ or 70° are higher (at least for the boundary conditions of Fig. 1, see below).

Fig. 2 shows a curve of the maximum growth rate as a function of the angle α between the plane of anisotropy and the single layer, for the same boundary conditions and material properties ($\mu_c = 10$, $\delta = 25$) as is in Fig. 1f. The expected reflection symmetry about the 45° direction is immediately obvious, with a maximum in the growth rate at $\alpha \approx 12.5°$ and 77.5°. These results indicate that, with free slip allowed on the side boundaries, the maximum initial growth rate of the single layer fold occurs when the planar anisotropy is only slightly oblique. However, the outcome is strongly influenced by the applied boundary conditions.

In Fig. 3, the effects of changing boundary conditions on the growth rate at very small fold amplitude are investigated, with a setup otherwise equivalent to Fig. 1f. For upper and lower model limits that are far removed from the central layer (for Figs. 1–3, the height of the model is eight times the width), a change in the upper and lower boundary conditions from (1) free slip in the x direction but prescribed v_y, to (2) totally prescribed v_x and v_y, has no effect on the growth rate. In run (3), the upper and lower boundaries were fully prescribed as in (2) but v_y was also set to zero at the inflection points on the mid-line of the initial sinusoidal perturbation in the single layer. This effectively ensures that the single layer itself cannot rotate. As can be seen from Fig. 3, this has no effect on the growth rates, even when the anisotropy in the matrix is oblique to the layer and to the boundaries (e.g. for $\alpha = 20°$ or 70°). In contrast, modifying the side boundary conditions does have a significant influence on the fold growth rate for orientations other than $\alpha = 0°, 45°$, or 90°. If, rather than allowing free slip, v_y on the sides is constrained to be periodic (by assigning only a single global degree of freedom in v_y to every pair of corresponding nodes on either side), the growth rate for $\alpha = 20°$ or 70° is significantly lower, as seen for (4) and (5) in Fig. 3, whereas there is no change for $\alpha = 0°, 45°$ or 90°. There is thus a markedly different response depending on whether the single layer alone is constrained to not rotate (no significant effect) or both the layer and anisotropic matrix are constrained to have no component of bulk rotation (leading to a reduction in growth rate).

In summary, the numerically calculated growth rate spectra show that in general the initial growth rates depend strongly on the orientation of the anisotropy plane with respect to the competent layer (Fig. 2), although the influence decreases for higher normal viscosity contrast between layer and matrix (Fig. 1). However, as was previously shown in Kocher et al. (2006), the geometry and kinematics of finite-amplitude folding is also influenced by matrix deformation processes, such as the formation of kink-bands or chevron folds, and these effects are not considered in the infinitesimal amplitude analytical solutions nor in the corresponding numerical models of Figs. 1–3.