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# Determination of relative strain ellipsoids from sectional measurements of stretching lineation

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**Abstract**

A novel strain inversion method is developed in this communication to determine relative strain ellipsoid (namely, principal directions and relative magnitudes) from four or more independent measurements of stretching lineation or the longest elliptical axes on planar surfaces. A linear, five-variable equation is obtained to describe such kind of measurements. Equations for these measurements are solved under some auxiliary constraints for the relative strain ellipsoid. This is similar in formulation to stress inversion. We believe the method will provide for structural geologists a simple, useful and more applicable tool for estimating strain in deformed rock where no passive strain marker other than stretching lineation is commonly observed at outcrop.


*Keywords:* Determination of finite strain; Relative strain ellipsoid; Sectional measurements; Stretching lineation

## 1. Introduction

For structural geologists, the determination of three-dimensional finite strain in rock is crucial in reconstructing the deformation history. Numerous graphic (e.g., Ramsay, 1967; Ramsay and Huber, 1983) and numerical (e.g., Shimamoto and Ikeda, 1976; Oertel, 1978; Milton, 1980; Gendzwill and Stauffer, 1981; Owens, 1984; Shao and Wang, 1984; Wheeler, 1989; De Paor, 1990; Robin, 2002; Shan et al., 2007) methods have been proposed for restoring the strain ellipsoid from strain ellipses measured on planar surfaces. In developing an inversion method recently for the same purpose, Shan et al. (2007) obtained a linear equation in six variables for each individual sectional measurement of stretching lineation or the longest elliptical axis, and highlighted an unresolved issue concerning the feasibility of using these equations to determine the strain ellipsoid. This issue is resolved in this short communication. As is proved below, this kind of strain measurement is sufficient for determining the relative strain ellipsoid (that is to say, principal directions and relative magnitude differences), but insufficient to determine absolute principal magnitudes.

A very similar idea can be derived from Tocher (1964) and others, who determined stereographically the optic axes of a crystal from a minimum of four independent extinction measurements, and was later applied to fabric analysis (Lisle, 1976). In essence, there is no difference in formulating sectional measurements of optic indicatrix, fabric ellipsoid and strain ellipsoid. However, a major contribution of this work is to render this concept in mathematics that will permit easy and fast use of a computer to make the determination.

Terms and their symbols used in this paper are listed in Table 1.

## 2. Presentation of the problem

Normally, a strain ellipsoid is considered in the Cartesian system (Fig. 1) as a quadric surface centered at the origin,

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Table 1
A list of symbols and their definitions

| Symbols | Definitions | Comments |
|---|---|---|
| $x$, $y$, and $z$ | Coordinates of a point on the ellipsoid in the real state. | Eq. (1). |
| $X$, $Y$, and $Z$ | Initial coordinate system. | |
| $x'$, $y'$, and $z'$ | Coordinates of a point on the ellipsoid in the rotated (reference) state. | Eqs. (3) and (4). |
| $X'$, $Y'$, and $Z'$ | Rotated coordinate system. | |
| $b_{ij}$ | Elements of a shape matrix. | $i, j$ = 1,2,3; Eqs. (1), (4)–(6) and (8)–(11). |
| $b_1$, $b_2$, and $b_3$ | Eigenvalues of a shape matrix. | $b_1 \geq b_2 \geq b_3 > 0$; Eq. (2). |
| $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$ | Magnitudes of the principal axes of an ellipsoid. | $\epsilon_1 \geq \epsilon_2 \geq \epsilon_3 > 0$; Eq. (2). |
| $\alpha$ and $\beta$ | Dip direction (azimuth) and dip angle of a certain measured planar surface. | Eq. (3). |
| $\theta$ | Pitch of the long axis of a strain ellipse on the planar surface. | Eq. (3). |
| $T$ | Inverse rotation matrix | Eq. (3). |
| $t_{ij}$ | Elements of inverse rotation matrix. | $i, j$ = 1,2,3; Eqs. (3)–(8) and (11). |
| $k$ | Elliptical parameter | Eq. (4). |

described by the following equation (e.g., Owens, 1984; Robin, 2002; Shan et al., 2007):

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1 \tag{1}$$

where $x$, $y$ and $z$ are the coordinates of a point on the ellipsoid in terms of the reference axes $X$, $Y$ and $Z$, and $b_{ij}$ ($i, j = 1, 2, 3$) is the element of the shape matrix (Shimamoto and Ikeda, 1976). The shape matrix is symmetrical, so $b_{ij} = b_{ji}$ ($i, j = 1, 2, 3$). The principal axes of the ellipsoid have the same directions as the eigenvectors of the shape matrix, but different dimensions from its eigenvalues,

$$\epsilon_1 = \frac{1}{\sqrt{b_3}}, \quad \epsilon_2 = \frac{1}{\sqrt{b_2}}, \quad \epsilon_3 = \frac{1}{\sqrt{b_1}} \tag{2}$$

where $b_1$, $b_2$, and $b_3$ are the corresponding eigenvalues of the shape matrix ($b_1 \geq b_2 \geq b_3 > 0$); $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$ are the corresponding principal radii of the ellipsoid ($\epsilon_1 \geq \epsilon_2 \geq \epsilon_3 > 0$).

Consider measurement of the direction of the longest elliptical axis, or stretching lineation, on a planar surface. We measure on the surface the dip direction ($\alpha$) and dip angle ($\beta$) of the surface, and the pitch ($\theta$) of the long axis of the sectional ellipse of the 3-D ellipsoid. The pitch is defined as the intersection angle between the long axis of the ellipse and the westward strike of the plane after the plane has been rotated around a vertical axis to dip toward the northward $X$-axis (Fig. 1).

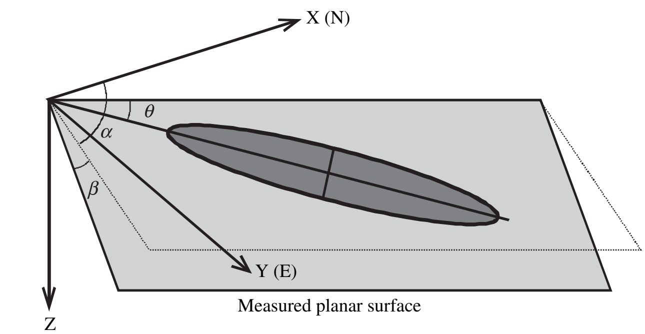


Fig. 1. Elements of strain measurement made on the planar surface in the Cartesian coordinate system (Shan et al., 2007). The $X$-axis is directed toward the north, the $Y$-axis toward the east, and the $Z$-axis upward. The blank rectangle, marked by three dashed lines and one thick line, represents a part of the horizontal ($X$–$Y$) plane. The gray rectangle marked by thick lines represents the plane where the stretching lineation is measured. See the text and Table 1 for symbol definitions.

For the given measurement, we will have a simple expression of the strain ellipse on the plane by manipulating a series of rotations to transform the strain measurement plane into a horizontal one where the long axis of the strain ellipse is aligned with the $X$-axis. This manipulation can be implemented by rotating around the $Z$-axis with an angle of $-\alpha$, around the $Y$-axis with an angle of $-\beta$, and finally around the $Z$-axis with an angle of $\theta - 90°$. Let $T$ stand for the inverse manipulation of these rotations that defines the relationship between the old and the new coordinate systems:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = T \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{3}$$

$$\begin{aligned} T &= \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \\ &= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \\ &\times \begin{bmatrix} \cos(90° - \theta) & \sin(90° - \theta) & 0 \\ -\sin(90° - \theta) & \cos(90° - \theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Application of the above transformation to Eq. (1) leads to the strain ellipsoid in its local $X'$–$Y'$–$Z'$ coordinate system. Because the strain ellipse of interest lies in the $X'$–$Y'$ plane after rotation, we have $Z' = 0$, thus giving the following expression for the rotated ellipse:

$$\begin{bmatrix} x' & y' \end{bmatrix} \begin{bmatrix} \begin{array}{l} t_{11}t_{11}b_{11} + 2t_{11}t_{21}b_{12} + 2t_{11}t_{31}b_{13} \\ +t_{21}t_{21}b_{22} + 2t_{21}t_{31}b_{23} + t_{31}t_{31}b_{33} \end{array} & \begin{array}{l} t_{11}t_{12}b_{11} + (t_{11}t_{22} + t_{21}t_{12})b_{12} \\ +(t_{11}t_{32} + t_{31}t_{12})b_{13} + t_{21}t_{22}b_{22} \\ +(t_{21}t_{32} + t_{31}t_{22})b_{23} + t_{31}t_{32}b_{33} \end{array} \\ \text{(symmetrical)} & \begin{array}{l} t_{12}t_{12}b_{11} + 2t_{12}t_{22}b_{12} + 2t_{12}t_{32}b_{13} \\ +t_{22}t_{22}b_{22} + 2t_{22}t_{32}b_{23} + t_{32}t_{32}b_{33} \end{array} \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = k \tag{4}$$