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An analytical solution in 2D for the motion of rigid elliptical particles with a slipping interface under a general deformation

Kieran F. Mulchrone*

Department of Applied Mathematics, National University of Ireland, University College, College Road, Cork, Ireland

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Abstract

A mathematical model for rigid inclusions with a slipping interface immersed in a general 2D homogeneous deformation is developed. Under bulk pure shear inclusions are expected to rapidly approach the stretching axis when compared to the behaviour of inclusions with no slip at the interface. The derived model predicts synthetic and antithetic motion into a stable orientation under simple shear, and thereafter the inclusion makes an antithetic angle with the shear direction. Under simple shear rotation rates can be higher or lower than those of no-slip inclusions, depending on orientation. A direct relationship between object inclination to the shear direction and the vorticity of the bulk flow is predicted. The model compares well with published analogue and numerical experiments.

Keywords: Slipping interface; Antithetic rotation; Stable direction; Ellipse

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1. Introduction

Natural rocks deform in very complex ways. Reliable estimates of finite strain and the kinematic vorticity number (W_k) or flow type are a key part of understanding local and regional rock deformation (see Xypolias and Koukouvelas, 2001 for an example study). A plethora of methods exist for finite strain estimation (see Mulchrone et al., 2003 and references therein), however, these rely largely on the assumption of passive behaviour (i.e. elliptical inclusions behave exactly like the enclosing matrix) and give no information about the kinematics of deformation. Over the last 20 years or so there has been a considerable research effort put into understanding porphyroclast systems (Passchier and Simpson, 1986) because they may be the source of kinematic and mechanical information (Bose and Marques, 2004). The mathematical model of Jeffery (1922) for the behaviour of a rigid ellipsoid immersed in a Newtonian fluid with no slip at the boundary has provided the theoretical basis for much of this work and was introduced to the structural geology literature by Ghosh and Ramberg (1976). More recently researchers have begun to consider different models for inclusion behaviour: (i) rigid with no-slip (Jeffery, 1922; Mushhelishvili, 1953; Mason and Manley, 1957; Bretherton, 1962; Ghosh and Ramberg, 1976; Freeman, 1985; Passchier, 1987; Jezek et al., 1999; Arbaret et al., 2001; Mandal et al., 2001; Schmid, 2002; Marques and Coelho, 2003), (ii) rigid with slip on the boundary (Ildefonse and Mancktelow, 1993; Odonne, 1994; Kenkmann and Dresen, 1998; Pennacchioni et al., 2000; Mancktelow et al., 2002; Schmid and Podladchikov, 2003, 2004; Samanta and Bhattacharyya, 2003; Ceriani et al., 2003; Bose and Marques, 2004; Marques and Bose, 2004; Marques et al., 2005a), (iii) rigid with slip and/or no-slip in confined flow (Marques and Coelho, 2001; Taborda et al., 2004; Marques et al., 2005b), (iv) non-rigid with no-slip (Eshelby, 1957; Bilby and Kolbuszewski, 1975; Schmid and Podladchikov, 2003; Mulchrone and Walsh, 2006). In this paper a 2D analytical solution is derived for the case of a rigid object immersed in a Newtonian fluid with slip at the boundary. After a brief review of related work, the solution is presented.

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* Tel.: +353 21 4903411; fax: +353 21 4271040.
E-mail address: k.mulchrone@ucc.ie

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Mantled porphyroclasts have been the subject of analogue modelling (Passchier and Sokoutis, 1993; Passchier et al., 1993; ten Brink and Passchier, 1995) and experimental results support a direct relationship between rheology and developed mantle structures (i.e. stress sensitivity). Theoretical and numerical studies of deflections around rigid spherical inclusions have also been conducted (Masuda and Mizuno, 1995, 1996a,b) for both Newtonian and non-Newtonian enclosing materials which indicated that it was the initial size of the mantle rather than the stress sensitivity which determined the type of mantle structure developed. Bons et al. (1997) showed that the applied boundary conditions (i.e. simple shear at an infinite versus a finite distance) determine the type of flow pattern around a rigid inclusion (i.e. eye-shaped or bow-tie-shaped separatrix, Passchier et al., 1993) and thus the mantle structure developed. Stress sensitivity was found to be of secondary importance. Mandal et al. (2001) demonstrated that a bow-tie separatrix can develop under combined simple and pure shear. Bose and Marques (2004) presented the results of analogue models suggesting that slip or no-slip at inclusion boundaries is an important factor in determining the morphology of mantle structures, as well as the flow pattern in the matrix, mantle rheology and the mantle position with respect to the separatrix. In a numerical study, Marques et al. (2005a) demonstrated the existence of cats eyes-shaped flow under the influence of a low viscosity layer between the matrix and inclusion. However, Schmid and Podladchikov (2005) used a three-phase finite-element model with power-law rheologies to investigate an isolated mantled porphyroclast in simple shear. They were able to produce gauges for effective mantle/matrix viscosity contrast, production rates of mantle material as a function of bulk shear strain and the total shear strain.

Populations of rigid inclusions have been the subject of theoretical (Fernandez et al., 1983; Fernandez, 1987; Passchier, 1987; Masuda et al., 1995; Marques and Coelho, 2003), experimental (Ildefonse et al., 1992a,b; Arbaret et al., 1996; Herweg and Handy, 1998) and natural study (Manga, 1998; Pennacchioni et al., 2001). In the case of populations of non-interacting particles (Fernandez et al., 1983; Fernandez, 1987; Masuda et al., 1995; Marques and Coelho, 2003) with no-slip at the boundary (Jeffery, 1922), pulsating fabrics have been predicted under simple shear although the period of a fabric cycle increases with inclusion aspect ratio. Interaction of particles (Ildefonse et al., 1992a,b; Arbaret et al., 1996) affects the behaviour of populations as revealed in analogue experiments. The development of pulsating fabrics is inhibited and the fabric remains at a small angle to the shear plane. Samanta et al. (2003) demonstrated theoretically and experimentally that for interacting spherical particles rotation rates are retarded.

In the vast majority of research no-slip boundary conditions are assumed, however, theoretical and experimental studies suggest that slip on the boundary between the inclusion and the matrix could significantly influence the behaviour of inclusions (Ildefonse and Mancktelow, 1993; Odonne, 1994; Kenkmann and Dresen, 1998; Pennacchioni et al., 2000; Mancktelow et al., 2002; Schmid and Podladchikov, 2003, 2004; Samanta and Bhattacharyya, 2003; Ceriani et al., 2003; Marques and Bose, 2004; Marques et al., 2005a,b). Ildefonse and Mancktelow (1993) observed increased inclusion rotation rates under simple shear but reduced rates under pure shear. Additionally, under simple shear they reported that inclusions rotated towards the shear plane and remained there. Due to modification of the soap layer used to facilitate slip, this conclusion is only valid for low finite strains. Furthermore, they found that because the inclusion does not rotate through the shear plane δ-type mantle structures will not develop. Odonne (1994) carried out analogue modelling of a deformable inclusion and found that with a high degree of bonding the inclusion deforms whereas as the level of bonding decreases the inclusion effectively behaves in a rigid manner. Marques and Coelho (2001) experimentally investigated the effect of simple shear applied at a finite distance on the behaviour of an isolated rigid inclusion and found that it departs quite significantly from that under simple shear applied at infinity and when the interface coupling is reduced antithetic rotation is possible. Mancktelow et al. (2002) performed ring shear experiments to investigate the effect of boundary slip or no-slip on differently shaped inclusions. Elliptical shapes with slip showed reduced rotation rates when the long axis is close to the shear plane, however, rhomboidal shapes with slip attained stable orientations with their long sides subparallel to the shear plane and back rotation (opposite to the bulk sense of shear) was also observed. In the case of no-slip, inclusions essentially behaved as predicted by the theory of Jeffery (1922). Ceriani et al. (2003) also describe antithetic rotation for elliptical inclusions and metastable positions for a lubricated interface. Metastable positions may be due to changing thickness of the layer of lubrication during deformation. In general, the presence of a lubricating mantle zone produces faster rotation rates than that predicted by the no-slip theory. Schmid and Podladchikov (2003) developed analytical solutions for the case of a deformable circular inclusion with a weak rim and have shown it may effectively be considered as a weak inclusion. Samanta and Bhattacharyya (2003) considered modes of detachment at a rigid-inclusion–matrix interface by calculating the stresses acting on the interface using Jeffery's (1922) theory (i.e. no-slip). They then studied the occurrence of detachments by initiating detachment once selected tensile and shear strengths had been exceeded. However, they did not investigate the influence of such detachments on the dynamics of clast behaviour. Bose and Marques (2004) and Marques and Bose (2004) reported on the results of precision experiments dealing with rigid inclusions composed of ice with slip on the boundary. There were clear differences between the behaviour of inclusions with slip and theoretical and experimental results for the no-slip case. In addition, they found antithetic rotation into a stable orientation which depends on aspect ratio and shape and also the occurrence of a metastable orientation separating the synthetic and antithetic rotational fields. A numerical study by Marques et al. (2005a), in which a low viscosity layer was placed between the matrix and inclusion, found close