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Hypoplastic simulation of normal faults without and with clay smears

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Abstract

The main objective of the present paper is to simulate the tectonic evolution of normal faults by means of hypoplasticity. This constitutive concept is introduced with state limits for sand-like and clay-like materials. Combined with conservation laws, and initial and boundary conditions, hypoplastic relations provide normal fault patterns, which are validated by model tests. For a cross section of the Lower Rhenish Basin with synsedimentary subsidence, the observed normal fault pattern is essentially reproduced. A realistic evolution of a single normal fault with the distortion of a clay layer into a clay smear is also obtained. The potential of our method for other tectonic simulations is indicated.

Animations of calculated evolutions are available in the Internet via http://www.ibf.uni-karlsruhe.de/material.html.

Keywords: Normal faults; Clay smears; Numerical simulation; Hypoplasticity

1. Introduction

There are striking similarities of soil deformations in a model box and tectonic deformations in the earth crust. This led to definitions of fault and shear systems, which are widely recognized in structural geology. Limit state plasticity concepts of soil mechanics are used for estimating stress fields from observed fault patterns (Mandl, 1988). This approach leaves open a number of questions:

- what is the influence of pressure and density,
- what is the role of time and viscosity,
- why do shear band patterns arise, and
- what kind of simulations can substitute model tests?

A key issue is the constitutive relations describing the mechanical behaviour of formation materials. Critical State Soil Mechanics (CSSM, Schoefield and Wroth, 1968) goes beyond conventional limit state plasticity. For critical states with stationary shearing, a pressure independent ratio of deviatoric and mean stress is proposed with a friction angle φ_c, and a void ratio e_c that decreases linearly with the logarithm of effective pressure p'. For isotropic first compression, a higher void ratio e_i depends likewise on p'. For higher than critical stress ratios, dilated shearing and the same decrease of e with higher p' is proposed. A lower bound void ratio was later proposed by Schoefield (2001) and related with tensile cracking. Various elastoplastic relations based on CSSM are used in geotechnical engineering.

Taking over CSSM for simulation of tectonic deformations would reveal some shortcomings: the linear e-log p' dependence can fail for big p', the lower e-bound can only be reached by decompression, and viscous effects are not allowed for. *Hypoplastic* relations imply state limits as asymptotic solutions (Gudehus, 1996), and were extended by polar terms for shear localizations (Tejchman and Gudehus, 2001). Such sand-like materials are introduced in Section 2. Clays are markedly rate-dependent as their particles are squashy, so they reveal creep and relaxation. Visco-plastic relations on the base of CSSM have been proposed for them (Adachi and Oka, 1982), but have as yet been scarcely used. Visco-hypoplastic relations (Niemunius, 2003) have a wider range of application. They represent clay-like materials and are introduced in Section 3.

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Normal faulting can be considered as a touchstone for the use of hypoplasticity for tectonic deformations. This is first shown in Section 4 for initially homogeneous formations. Swarms of normal shear bands have been observed in model sand boxes, e.g. Wolf et al. (2003). We obtain such realistic shear band patterns that they can substitute model tests and provide more insight. For field-size problems finite element mesh sizes down to a few grain diameters are neither feasible nor necessary. For a known case of synsedimentary subsidence, realistic swarms of normal and antithetic faults are obtained.

A mechanism for the evolution of *clay smears* in normal faults was proposed by Lehner and Pilaar (1995). They postulate a viscous extrusion by a pressure gradient from the far-field towards the fault. As shown in Section 5, we obtain a combination of extrusion and shearing and a pressure gradient towards the fault. Furthermore, we obtain an asymptic clay smear thickness and a continued widening of the fault, and increasing swarms of antithetic shears. This is validated by observations (Weber et al., 1978).

Conclusions and an outlook are given in Section 6. It appears that the questions raised at the onset of this Introduction can be answered more and more by means of hypoplastic simulations. Tectonic deformations more complex than normal faulting could also be grasped.

2. Hypoplastic behaviour of psammoids

A *psammoid* (Greek psammos = sand) is an idealized substitute of a granular soil—like silt, sand or gravel. Granular permanence is assumed, i.e. changes of grains except for plastic flats at their contacts are neglected. Three limit void ratios depend on the mean solid partial pressure p_s as shown in Fig. 1a. The upper bound e_i denotes e for an isotropic compression starting from a very loose state, e_c is the critical void ratio for stationary shearing, and e_d denotes a lower bound approached by cyclic shearing with constant p_s and small strain amplitudes.

The curves of Fig. 1a are described (Bauer, 1996) by

$$\frac{e_i}{e_{io}} = \frac{e_c}{e_{co}} = \frac{e_d}{e_{do}} = \exp\left[-\left(\frac{3p_s}{h_c}\right)^n\right]. \quad (1)$$

The granular hardness h_s ranges from ca. 10 MPa to 10 GPa, and the exponent n from ca. 0.3 to 0.6. The prefactors e_{io}, e_{co} and e_{do} have nearly constant ratios, $e_{io}/e_{co} \approx 1.1$ and $e_{do}/e_{co} \approx 0.6$, so only e_{co} is needed. A relative void ratio

$$r_e = (e - e_c)/(e - e_d), \quad (2)$$

is defined by the actual e and the p_s-dependent limit values e_c and e_d.

The stress state of the grain skeleton is described by a tensor. With cylindrical symmetry, like in a triaxial apparatus, axial and radial components σ_{s1} and $\sigma_{s2} = \sigma_{s3}$ can be represented in a plane (Fig. 1b), and substituted by $p_s = (\sigma_{s1} + 2\sigma_{s2})/3$ and a deviatoric angle $\psi_\sigma = \text{atan}\,\sqrt{2}(\sigma_{s1} - \sigma_{s2})/3p_s$. Hypoplastic relations express skeleton stress rates as functions of stress, void ratio and strain rate, viz.

$$\dot{\sigma}_{si} = f_s\left(L_{ij}\varepsilon_j + r_e^\alpha N_i D\right), \quad (3)$$

with summation in j and $i = 1,2$ for cylindrical symmetry. $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2 = \dot{\varepsilon}_3$ can be substituted by the rate of volume change $\dot{\varepsilon}_v = \dot{\varepsilon}_1 + 2\dot{\varepsilon}_2$, and a direction angle $\psi_{\varepsilon} = \text{atan}\,\sqrt{2}(\dot{\varepsilon}_1 - \dot{\varepsilon}_2)/\dot{\varepsilon}_v$, Fig. 1c. $D = \sqrt{\dot{\varepsilon}_1^2 + 2\dot{\varepsilon}_2^2}$ denotes the modulus of strain rate. L_{ij} and N_i are functions of ψ_σ and the critical friction angle φ_c. f_s is determined by Eq. (1) and therefore proportional to h_s and $(\rho_s/h_s)^{1-n}$. For state limits, simple relations hold between $\psi_{\dot{\varepsilon}}$, ψ_σ and r_e, Fig. 1d and e. For an isotropic compression from $e = e_{io'}$ (i), $\psi_\sigma = \psi_{\dot{\varepsilon}} = 0$ holds with $r_e = (e_{io} - e_{co})/(e_{co} - e_{do})$. For stationary shearing at critical states, i.e. $\psi_{\dot{\varepsilon}} = \pm 90^\circ$ and $\dot{\sigma}_{si} = 0$ the stress condition reads

$$(\sigma_{s1} - \sigma_{s2})^2/(\sigma_{s1} + \sigma_{s2})^2 = \sin^2\varphi_c. \quad (4)$$

This means two different ψ_σ for axial shortening and stretching (c and −c). $e = e_c$ means $r_e = 1$. Axial splitting with $\sigma_{s2} \to 0$ and $\dot{\varepsilon}_1 \to 0$ and discing with $\sigma_{s1} = 0$ and $\dot{\varepsilon}_2 \to 0$ are unattainable extreme cases with $r_e = 0$ (d and −d). For peak states in between critical states and splitting Eq. (4) holds with a peak friction angle φ_p instead of φ_c and with dilation.

State limits are asymptotic solutions of Eq. (3) for a given $\psi_{\dot{\varepsilon}}$ and can therefore be called *attractors* (whereas they are assumed *a priori* in elastoplastic relations). This concept works also for other than cylindrically symmetric deformations, in particular simple shearing, and has been validated (Gudehus, 2006). It implies *shear localization* for peak states in two conjugate planes inclined by ca. $45^\circ - \varphi_p/2$ against the σ_{s1}-axis for cylindrical shortening. Splitting or discing may be interpreted as anomalous shear localization with $\varphi_p = 90^\circ$.

Evolutions of state and shape of representative volume elements (RVE) can be simulated numerically with the hypoplastic relation (3). An initial state has to be assumed within the allowed range of stresses and void ratios. Two of the four components σ_{s1}, σ_{s2}, ε_1 and ε_2 can be given as time-dependent boundary conditions, and two can be calculated using Eq. (3). Other than with elastoplastic relations, strength and stiffness are thus derived and not assumed *a priori*.

Shear bands arise throughout an RVE beyond a peak state so that it is no more homogeneous. This can be modelled by hypoplasticity with finite elements and different sophistications, e.g. for stretching of a strip between two smooth plates with constant lateral pressure (Fig. 2a). A zig-zag pattern with reflections at the plates is obtained by means of Eq. (3) if $e < e_c$ holds at the onset. e tends to e_c in the bands; in the end, the whole strip attains a critical state. One can approximate the overall behaviour using Eq. (4) with a peak friction angle φ_p instead of φ_c and with spatial averages $\overline{\sigma}_{s1}, \overline{\sigma}_{s2}$ and $\overline{e}$ of the state variables.

These findings are partly mesh dependent. Peak stress ratios and shear band inclinations can be obtained realistically if the mesh is not too coarse. However, the width of primary shear bands is determined by the element size, and more so the subsequent evolution. This lack of objectivity can be overcome by polar quantities (Gudehus and Nübel, 2004). The shear band width is restricted to ca. 5–15 grain diameters by the