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Buckling folds of a single layer embedded in matrix – Folding behavior revealed by numerical analysis

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ABSTRACT

Theoretical solutions have been proposed recently for various competent layer–matrix systems, including elastic, viscous and elasto-viscous materials. Furthermore, three type fold-forms of buckling fold had been proposed. These solutions were obtained based on the most simplified, one-dimensional governing equations. Therefore, these solutions require further validation by observing the two-dimensional folding behaviors. This work utilizes numerical analyses to study the buckling and post-buckling behaviors of various layer–matrix systems. As a result, it was found that for competence contrast $R \geqq 10$ the fold-forms obtained by numerical simulation agree well with those theoretical solutions. Three types of fold-forms can be generated and the resulting wavelengths are also close to the predictions. The fold evolution during the post-buckling stage is explored up to high amplitudes, and the results indicate that the fold-forms can remain the same or be changed from one type to another type, depending on the types of layer–matrix system, the applied strain rates, the original fold-forms at buckling, etc. The fold behaviors from buckling to the post-buckling stage of the layer–matrix systems are presented.

1. Introduction

For a stiff, competent layer of rock stratum embedded in relatively softer matrix, the lateral compression of this competent layer–matrix system can induce folding of the rock stratum and surrounding matrix, which is often referred as buckle-folding. Research on buckle-folding can be dated to the early 1900s (Smoluchowski, 1909). Field observations and interpretations focused on formation, wavelength and thickness have been made regarding the appearance of folds and mechanism (Sherwin and Chapple, 1968; Donath and Parker, 1964; Hudleston, 1986; Hudleston and Lan, 1993). To further explore the mechanism, studies based on laboratorial experiments to explore evolution have been conducted (Biot et al., 1961; Hudleston, 1973b; Fletcher, 1974; Dubey and Cobbold, 1977; Abbassi and Mancktelow, 1990, 1992; Mancktelow and Abbassi, 1992; Treagus and Sokoutis, 1992). Theoretical solutions considering various material types (elastic, viscoelastic and viscous) have accordingly been developed (Karman and Biot, 1940; Biot, 1957, 1959, 1961; Currie et al., 1962; Ramberg, 1961, 1963, 1964; Chapple, 1968; Smith, 1975, 1977, 1979; Jeng et al., 2001; Jeng and Huang, 2008), and the fold formation in deformed layers are considered according to the competence contrast between layer and matrix. Studies considered in more sophisticated conditions have also been conducted, e.g. for single layer systems (Chapple, 1969; Cobbold, 1975, 1976, 1977; Fletcher, 1974, 1977; Hudleston, 1973a,b; Hudleston and Stephansson, 1973; Hudleston and Lan, 1994; Hunt et al., 1996a,b; Kocher et al., 2008; Lan and Hudleston, 1991, 1996; Mancktelow, 1999; Mühlhaus et al., 1994; Schmalholz and Podladchikov, 1999, 2000; Treagus, 1973; Williams et al., 1977; Zhang et al., 1996, 2000); for stress–strain analysis (Dieterich and Carter, 1969; Hobbs, 1971; Hudleston et al., 1996; Treagus, 1981, 1983, 1999, 2003); for bending folds (Latham, 1985a,b); for deformation rate (Price, 1975; Johnson and Fletcher, 1994; Mühlhaus et al., 1998, 2002a,b; Schmalholz and Podladchikov, 2001a,b,c; Treagus, 2003); for heterogeneous deformation (Passchier et al., 2005); for nonperiodic folds (Whiting and Hunt, 1997) and a brief summary is given by Price and Cosgrove (1990). In these studies, folding with a single wavelength, the so-called dominant wavelength, has been recognized.

In addition to the findings of previous researches, folds with dual wavelength or decaying amplitude were found possible, when considering the solutions in a more general manner (Mühlhaus et al., 1998; Jeng et al., 2001, 2002; Jeng and Huang, 2008; Hobbs

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Table 1
$\varepsilon_x^B - l$ relationships of three fold-forms, expressed in terms of R_E and R_V, for various models (Jeng and Huang, 2008).

	Material layer–matrix	Fold-form*		
		Type A	*Type B* (l_d)	*Type C*
1.	*E–E*	$\varepsilon_x^B = (\pi^2/3l^2) + (l/2R_E\pi)$	$l_d = 2\pi\sqrt[3]{R_E/6}$	$\varepsilon_x^B = (2\pi^2/l^2) - \sqrt{3\pi/4R_E l}$
2.	*V–V*	$\varepsilon_x^B = (\pi^2/3l^2) + (l/2R_V\pi)$	$l_d = 2\pi\sqrt[3]{R_V/6}$	$\varepsilon_x^B = (2\pi^2/l^2) - \sqrt{3\pi/4R_V l}$
3.	*EV–EV*	$\varepsilon_x^B = (\pi^2/3l^2) + (l(1 - e^{-T/T_{Ro}}))/(2R_V\pi^{(1 - e - (R_E/R_V)(T/T_{Ro})}))$	$l_d = 2\pi\sqrt[3]{R_V/6}[(1 - e^{-(R_E/R_V)(T/T_{R0})})/(1 - e^{-(T/T_{R0})})]^{1/3}$	$\varepsilon_x^B = (2\pi^2/l^2) - \sqrt{(3\pi/4R_V l)((1 - e^{-(T/T_{R0})})/(1 - e^{-(R_E/R_V)(T/T_R)}))}$
4.	*E–EV*	$\varepsilon_x^B = \pi^2/3l^2 + (l/\overline{R}_E(T/T_{R0})\pi)(1 - e^{-(T/T_{R0})})$	$l_d = 2\pi\sqrt[3]{\overline{R}_E(T/T_{R0})/12}[1/(1 - e^{-(T/T_{R0})})]^{1/3}$	$\varepsilon_x^B = 2\pi^2/l^2 - \sqrt{3\pi(1 - e^{-(T/T_{R0})})/2\overline{R}_E l(T/T_{R0})}$
5.	*EV–V*	$\varepsilon_x^B = \pi^2/3l^2 + (l/2R_V\pi(1 - e^{-(T/T_R)}))$	$l_d = 2\pi\sqrt[3]{R_V/6}[1 - e^{-(T/T_R)}]^{1/3}$	$\varepsilon_x^B = 2\pi^2/l^2 - \sqrt{3\pi/4R_V l(1 - e^{-(T/T_R)})}$
6.	*E–V*	$\varepsilon_x^B = \pi^2/3l^2 + (l/\pi(T/(T_R)_{E-V}))$	$l_d = 2\pi\sqrt[3]{(1/12)(T/(T_R)_{E-V})}$	$\varepsilon_x^B = 2\pi^2/l^2 - \sqrt{3\pi/2l(T/(T_R)_{E-V})}$

*Remarks:
1. $R_E = \overline{E}/\overline{E}_o = \overline{G}/\overline{G}_o$; $\overline{R}_E = \overline{G}/G_o$; $R_V = \eta/\eta_o$; $T_R = \eta/G$; $T_{Ro} = \eta_o/G_o$; $(T_R)_{E-V} = \eta_o/\overline{G}$.
2. The symbols and their definitions please refer to Jeng and Huang (2008).

et al., 2008). Furthermore, it is necessary to consider that the material properties of the competent layer and the matrix may change with the local environments e.g. temperature and pressure conditions. These conditions can be simply expressed in elastic, viscoelastic and viscous materials. The three types of material can have six kinds of possible combinations for real geological conditions. Two-component notations are used hereafter to indicate the layer–matrix system. For instance, the notations *E–E*, *E–EV*, *E–V* represent the cases of an elastic layer respectively embedded in the elastic, viscoelastic and viscous matrix. The solutions for six possible models (*E–E*, *E–EV*, *E–V*, *EV–EV*, *EV–V* and *V–V*) have been established (Biot, 1961; Currie et al., 1962; Jeng and Huang, 2008).

It was identified that the resulting waveforms depend on the amount of compression, and could be expressed in terms of lateral force or lateral strain in rock layer, when bucking (or folding) occurs. The relation of the lateral compression at the moment of buckling (ε_x^B) and the resulting wavelength l for the six aforementioned models is conveniently summarized in Table 1 (Biot, 1961; Currie et al., 1962; Jeng and Huang, 2008). In general, the $\varepsilon_x^B - l$ relationship has the typical appearance as shown in the left-side of Fig. 1. The upper curve is related to the fold-form with dual-frequencies and the lower curve is the fold-form with single frequency yet with decaying amplitude. The intersection of the upper and the lower curves is the *Type B* fold-form, which is also referred as fold-form at critical state (Biot, 1961; Currie et al., 1962). These three types of fold-form are depicted in the right-side of Fig. 1. Among these six models, *E–E* and *V–V* models are found to be strain rate-independent and the other four models are strain rate-dependent (Jeng and Huang, 2008). For convenience, Table 2 summarizes the response of the six models when subjected to extreme strain rates.

In developing theoretical solutions for the six models, two-dimensional folding phenomena are simplified as one-dimensional governing equations (Biot, 1961; Currie et al., 1962; Hunt et al., 1996a,b; Schmalholz and Podladchikov, 1999, 2000; Jeng and Huang, 2008). Therefore, there is still a suspicion that the one-dimensional governing equations yielding solutions may not be adequate to simulate two-dimensional buckling fold-forms. Moreover, the governing equation is the state of force (or stress) equilibrium at the moment of buckling and thus the solutions can only represent the fold-form at the moment of buckling. The folding evolution throughout entire deformation history after the buckling stage is important. Thus, it is of interest to ask what will happen after the buckling? Can the fold-form maintain same wavelength? Or, will the wavelength, even the type of fold-form, be changed after the folds have been generated, during the post-buckling stage of deformation? For the sake of convenience, the deforming process after the fold initiation is called post-buckling stage.

Previous research indicated that numerical simulations based on finite element method yielded reasonable fold-forms. The resulting dominant fold-form by numerical simulation with high competence contrast agreed with the theoretical solution (Zhang et al., 1996, 2000; Mancktelow, 1999, 2001; Jeng et al., 2002). As to other models (*E–EV*, *EV–EV*, *EV–V*, *E–V*), comparisons are not completely made because some solutions have been only recently proposed (Jeng and Huang, 2008). This research aimed at exploring the above-mentioned questions based on two-dimensional numerical analyses. The observations focus on: What are the types of fold-form corresponding to different degrees of lateral compression? This also means: can three types of fold-form really happen in numerical simulation? How well do the resulting wavelengths agree with the theoretical solutions as summarized in Table 1? Are these models really rate-dependent or rate-independent as described by the theoretical solutions? What is the