ELSEVIER

Journal of Structural Geology 29 (2007) 1558–1570

JOURNAL OF STRUCTURAL GEOLOGY

www.elsevier.com/locate/jsg

Shape fabrics in populations of rigid objects in 2D: Estimating finite strain and vorticity

Kieran F. Mulchrone *

Department of Applied Mathematics, National University of Ireland, Cork, Ireland

Received 29 June 2006; received in revised form 6 June 2007; accepted 23 June 2007
Available online 18 July 2007

Abstract

Shape fabrics of elliptical objects in rocks are usually assumed to develop by passive behaviour of inclusions together with the surrounding material leading to shape-based strain analysis methods belonging to the R_f/ϕ family. By deriving the probability density function for populations of rigid ellipses deforming in a general 2D deformation, a method is developed which can be used to estimate both finite strain and the kinematic vorticity number. Statistical parameters are theoretically derived and their behaviours under various kinematic conditions are investigated. The maximum likelihood method from statistics is used to produce a numerical method for estimating deformation parameters from natural populations. A simulation study demonstrates that finite strain can be estimated well for both low and high applied finite strains, whereas the kinematic vorticity number is well estimated only in the case of high finite strains ($R_s > 40$), and that large sample numbers (≈ 1000) are required.

Keywords: Deformation; Shape fabric; Kinematic vorticity number; Rigid object; Probability distribution function

1. Introduction

Shape fabrics are a common feature of deformed rocks and are usually defined by elongate, approximately elliptical, clasts or porphyroblasts. The finite strain which leads to a shape fabric is usually estimated by applying the R_f/ϕ and related methods of strain analysis. A shared assumption of all these methods is that individual markers behaved passively during deformation. There are many instances where this is not the case and application of such methods can lead to erroneous results. Therefore the purpose of this paper is to develop a method which allows calculation of finite strain from a population of rigid objects. Additionally, as has been noted by previous workers (e.g. Masuda et al., 1995; Marques and Coelho, 2003), rigid object populations have the potential to allow for estimation of the flow type (i.e. kinematic vorticity number) that produced a particular distribution. Thus, the method developed in this paper may also be used to estimate the flow vorticity.

There has been considerable interest in the properties of populations rigid objects modified during deformation, most of which is underpinned by the landmark work of Jeffery (1922). Experimental work, both analogue and numerical, has served to advance our understanding of the behaviour of interacting populations of objects (see for example Ildefonse et al., 1992a,b; Arbaret et al., 1996; Jezek et al., 1996, 1999; Piazolo et al., 2002; Mulchrone et al., 2005). Fernandez et al. (1983, 1987) derived the theory of rigid object shape fabrics developed under 2D simple shear and observed the cyclical nature of such fabrics. Masuda et al. (1995) conducted an essentially numerical study whereby initially uniform distributions of rigid particles were deformed over a range of axial ratios and kinematic conditions. They found a gradual transition from symmetric to asymmetric distributions (when considered across all axial ratios) as the kinematics went from pure to simple shear, respectively. Recently, Marques and Coelho (2003) extended our understanding of the behaviour of rigid object populations by examining their behaviour under

* Tel.: +353 21 4903411.
E-mail address: k.mulchrone@ucc.ie

0191-8141/$ - see front matter
doi:10.1016/j.jsg.2007.06.006

transtensional and transpressional regimes. This was achieved by deriving analytical expressions for object orientations over time and plotting solutions for a variety of initial orientations.

In this paper, the theoretical distribution of rigid object populations as a function of both axial ratio and deformation regime is derived for a general 2D deformation. A method is suggested which allows for estimation of the finite strain and kinematics of the responsible deformation. It is important to acknowledge the limitations of the approach at the outset which are directly related to the mathematical models utilised. Firstly, the method derived is valid only for the 2D case, and in contrast to typical R_f/ϕ methods, 3D effects may be important in practice. In other words, identical 2D elliptical sections through differently shaped ellipsoids will behave differently. Secondly, inherent in Jeffery's (1922) model for rigid particle motion, the particle is isolated and interaction effects are not taken into consideration. Interaction is likely to be an important component of natural behaviour of particle populations.

2. Flow kinematics, rigid object rotation and finite strain evolution

The modelling approach of Mulchrone et al. (2005) is closely followed and many of the details are therefore omitted. Important equations are listed and some derivations regarding the finite strain state are presented. Homogeneous flows described by the velocity gradient tensor:

$$\mathbf{L} = \begin{pmatrix} 0 & L_{12} \\ L_{21} & 0 \end{pmatrix} \quad (1)$$

are considered. This flow is automatically incompressible and admits all general deformations recognised by Ramberg (1975), Means et al. (1980) and Ghosh (1987) and the kinematic vorticity number is given by:

$$W_k = \frac{L_{12} - L_{21}}{|L_{12} + L_{21}|} \quad (2)$$

The convention for angles and rotations used here is that the positive ordinate axis is the zero angle direction and the counter-clockwise angles and rotations are positive and vice versa for the clockwise case.

The eigenvectors of **L** (also known as the flow apophyses by Ramberg, 1975) are directions of zero rotation, i.e. material particles initially within these directions remain so, whereas other particles tend to be repelled from or asymptotically attracted into the flow apophyses (Passchier, 1997). They are particularly helpful in visualising the passive behaviour of points and lines as they divide the flow into zones of counter-clockwise and clockwise rotation (see Fig. 1). Taking $\xi = \sqrt{L_{12}/L_{21}}$, the eigenvectors are given by:

$$l_1 = \begin{pmatrix} \xi \\ 1 \end{pmatrix} \text{ and } l_2 = \begin{pmatrix} -\xi \\ 1 \end{pmatrix} \quad (3)$$

and have orientations:

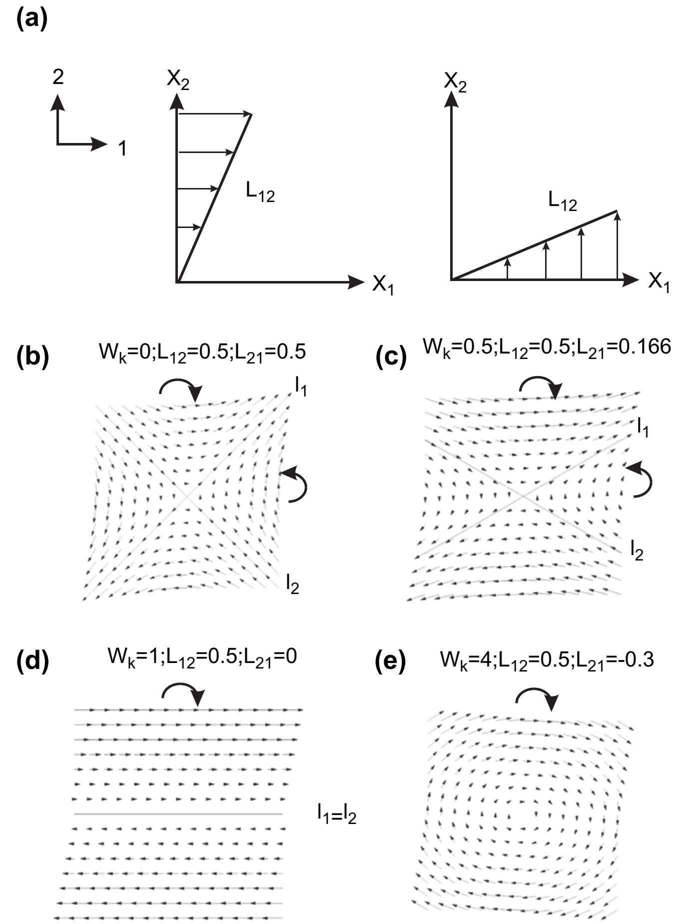

Fig. 1. (a) Velocity gradient tensor components L_{12} and L_{21} describe how the velocity vectors (small arrows) linearly vary near the origin. Combining these components in different proportions generates all possible flow types. For example, (b) $W_k = 0$, pure shear with two mutually normal flow apophyses (l_1 and l_2). Heavy arrows show the local rotation sense. (c) $W_k = 0.5$, intermediate between pure and simple shear, with two sub-normal flow apophyses. (d) $W_k = 1$, simple shear both flow apophyses coincide along the shear direction and (e) $W_k = 4$, no real apophyses exist and pulsating strain histories occur.

$$\theta_1 = \tan^{-1}\left(\frac{1}{\xi}\right) \text{ and } \theta_2 = -\tan^{-1}\left(\frac{1}{\xi}\right)$$

It is clear that for $|W_k| > 1$, some of the components of the eigenvectors are imaginary and do not have a physical meaning. The two eigenvectors coincide under simple shear ($|W_k| = 1$) and in all other cases remain distinct.

Under a general deformation of the type described by Eq. (1), Mulchrone et al. (2005) have shown that a rigid elliptical object rotates at a rate given by:

$$\dot{\phi} = \frac{L_{21} - L_{12}}{2} + \frac{(L_{21} + L_{12})(R^2 - 1)}{2(R^2 + 1)}\cos 2\phi \quad (4)$$

where ϕ is the orientation of the major axis of the object and R is the axial ratio. This equation is used extensively in the next section.

As one of the aims of this paper is to explore the relationship between rigid object fabrics and the strain ellipse, equations are derived that describe the evolution of the strain