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Visualizing strain and the R_f–Φ method with an interactive computer program

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ABSTRACT

The R_f–Φ method is a powerful graphical approach for estimating finite strain of deformed elliptical objects, but one that students commonly find difficult to understand. We developed a program that allows users to explore visually how deforming a set of elliptical objects appears on R_f–Φ plots. A user creates or loads the ellipses and then deforms them by simple shear, pure shear, or rigid rotation. As the ratio of the long to short axis of the ellipses (R_f) and long-axis orientations (Φ) change in one window, the R_f–Φ plot *continuously and instantaneously* updates in another. Users can save snapshots of the deformed elliptical objects and the R_f–Φ plots to record graphical experiments. The program provides both R_f vs. Φ and polar ln(R_f) vs. 2(Φ) plots. The user can 'undeform' ellipses quickly and easily, making it possible to inspect the 'original' shapes and orientations of objects, and to evaluate the plausibility of the determined strain values. Users can export information about the pebbles' shape and orientation to spreadsheets for rigorous statistical analysis. This program is written in Java and so can run on virtually any operating system. Both the source code and the application will be freely available for academic purposes.

1. Introduction

Deformed ellipsoidal objects, such as pebbles and oolites, are common in rocks, and they offer an intuitive, visually appealing approach for teaching fundamental strain concepts in structural geology. Students can easily grasp the effect of strain on an initially spherical object and, with practice, can visualize the fate of initially ellipsoidal markers. Furthermore, the study of deformed pebble conglomerate and oolitic limestone provides an excellent opportunity for students to gain experience in data acquisition and error analysis. They must also confront a host of important problems that plague all attempts to quantify strain, such as ductility contrast between marker object and matrix, initial shape and distribution of marker objects, area or volume change during deformation, and the relationship between two-dimensional strain measured in planar sections with the three-dimensional strain experienced by rocks.

Structural geologists commonly exploit elliptical objects in their research to quantify strain in naturally deformed rocks, understand the development of deformation fabrics, and examine strain gradients in folds and fault zones. The importance of this approach to strain measurement has inspired numerous studies to overcome its inherent limitations, or at least to understand them thoroughly. Through numerical experiments, Lisle (1979) tested several methods for averaging shape and orientation data to determine the most accurate for estimating the strain ellipse, and concluded that the harmonic mean was the most reliable. Hossack (1968) and Treagus and Treagus (2002) also discussed in detail the problems of determining strain from pebble shapes in a conglomerate.

Ramsay (1967) derived the equations of the R_f–Φ method for quantifying finite strain, and Dunnet (1969) showed how the R_f–Φ method can be used as a practical tool for strain determination from elliptical objects. As discussed in detail below, the R_f–Φ method assumes an initially random distribution of ellipse long-axis orientations, and a range of initial long to short axial ratios, R_i (Table 1). Ramsay and Huber (1983) presented an especially useful, and well-illustrated, discussion of the R_f–Φ method making the technique more accessible to researchers and students. Lisle (1985) offered a very complete and useful treatment of the method. Our contribution is to provide a program that links deformation of elliptical objects with R_f–Φ plots, giving students a *visual* explanation of how the method works, and offering students and researchers a tool to quickly estimate strain from outcrops and samples.

Our program provides both the familiar Cartesian R_f–Φ plot and the innovative polar plot of Elliott (1970) for comparison with the ellipse population. Elliott's (1970) approach employed a novel "shape factor grid" and a polar plot of ln(R_f) vs. 2(Φ) that should, in theory, allow assessment of the initial distribution of long axes of

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Table 1
Abbreviations used in text and figures

Φ	The angle between an arbitrary reference line and the long axis of an ellipse. Range: $-90°$–$90°$.
R_f	The final axial ratio (long axis/short axis) of any arbitrary elliptical object.
R_{fmax}	The maximum final axial ratio of all the elliptical objects.
R_{fmin}	The minimum final axial ratio of all the elliptical objects.
R_i	The initial axial ratio of any arbitrary elliptical object.
R_{imax}	The maximum initial axial ratio of all the elliptical objects.
R_s	The axial ratio of the strain ellipse (long axis/short axis).

elliptical objects, and estimate the strain. The greatest limitation of this approach is the apparent complexity of the distribution of undeformed elliptical objects (Boulter, 1976; Paterson and Yu, 1994). Yamaji (2005) developed an inverse method to overcome some of these limitations for the special case of a bivariate normal distribution of sedimentary particles. Another limitation to Elliott's (1970) approach, and a possible explanation for why the method has been underutilized, is the significant difficulty most users have with visualizing the effect of strain on elliptical objects in polar plots. Our program helps overcome the latter limitation by showing how Cartesian and polar R_f–Φ plots change during deformation.

Lisle's (1985) approach to testing the assumption of an initially random distribution of pebble long axes was to create a set of "marker deformation grids" using Cartesian R_f–Φ plots, and to examine the distribution pattern of deformed ellipses. Sets of pebbles that were consistent with the assumption should show a symmetrical pattern about both the harmonic mean of R_f and the vector mean of Φ. De Paor (1988) developed another novel and useful approach to the R_f–Φ method that uses a hyperbolic net and symmetry principles to estimate strain from ellipsoids, but our program does not include hyperbolic plots.

Several commercially available drafting programs allow users to create a set of elliptical objects and to simulate deformation with tools that linearly transform the ellipses by pure shear, simple shear, and rigid rotation. These programs are very useful for teaching purposes. Also, several commercial programs are available that permit researchers to determine strain with the R_f–Φ method using axial ratio and orientation data. Some of these programs incorporate statistical methods to assess the validity of the assumption of initial random distribution of long-axis orientations. We have not duplicated the capabilities of these programs. Instead, we developed a relatively simple program that focuses on *visualizing* the relationship between strained elliptical objects and plots of axial ratio vs. orientation. Our program is complimentary to the existing software because it is straightforward to examine deformed objects with our program and then, to export information about the ellipses to data files for use in these programs.

Our goal was to create a simple and easy-to-learn interactive computer program that allows the user to simulate deformation of elliptical objects by pure shear, simple shear, and rigid rotation. Throughout the linear transformations, Cartesian or polar R_f–Φ plots are continuously and instantaneously updated. The advantage of this program is that the color coding and tracking options make it possible to visualize the distribution *and paths* of points representing elliptical objects on R_f–Φ plots. This is especially valuable for the polar plots of $\ln(R_f)$ vs. $2(\Phi)$, and it helps highlight the potential of this neglected approach. We also show here how the program is used to introduce students to the R_f–Φ method, determine strain in natural samples, and simulate 'retro-deformation' of samples to recover the original shapes and orientation of the pebbles for critical evaluation of the method. This program is written in Java, and so can run on virtually any operating system. Both the source code and the finished application will be freely available for academic purposes.

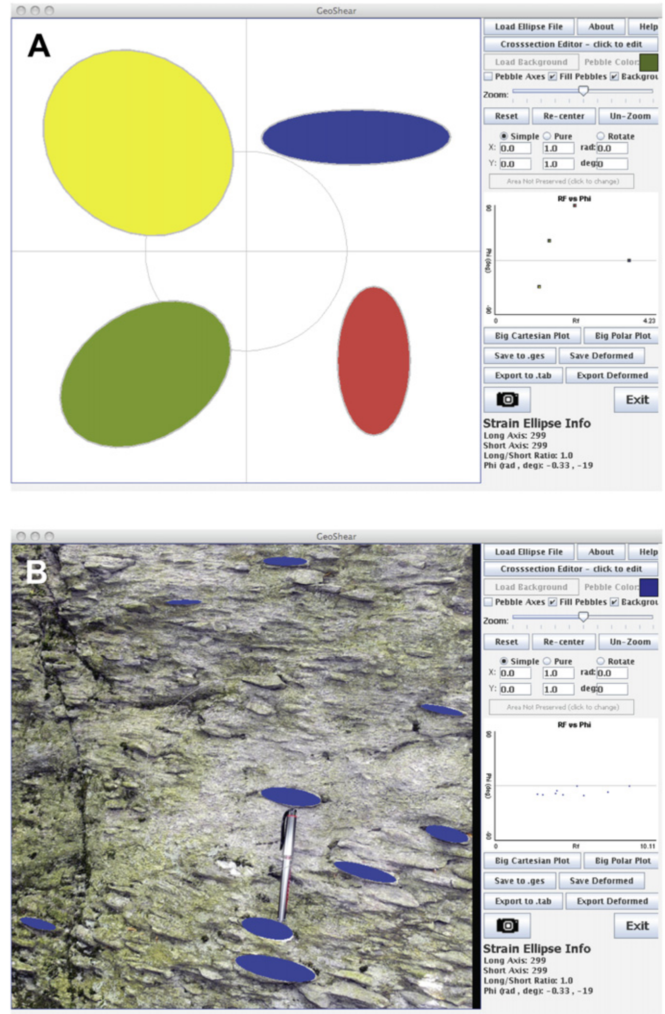

Fig. 1. Screen captures of the entire program window. **A.** Individual ellipses can be drawn in editing mode with a wide variety of colors. **B.** Photographs (up to 700 by 700 pixels) of deformed objects, such as these quartz pebbles, are imported in the display window for tracing ellipses as shown in blue.

2. Summary of the program

The program contains a large display window on the left and a display control window on the right (Fig. 1A). Ellipses are created in the display area by dragging with the mouse in editing mode. The user can import a 700 by 700 pixel photograph or other image as a background, and trace elliptical objects from it (Fig. 1B). Alternatively, text files containing information about the position, shape, and orientation of elliptical objects can be loaded. An Excel Workbook that serves as a template for creating such files is included with the program. Several buttons control the appearance of the ellipses and the scale and position of the display (Table 2). All pebbles in the display area are plotted in the small R_f vs. Φ plot in the display control window as they are created (Fig. 1A).

The user can choose deformation by simple shear, pure shear, or rigid rotation with the radio buttons (Fig. 1). Once a radio button is chosen, deformation can be precisely specified or accomplished by click and drag with the mouse in the display area. During simulated deformation, the R_f vs. Φ plot is continuously and instantaneously updated as ellipses change shape and orientation in the display area. A larger and more versatile R_f vs. Φ plot appears when the user clicks the Big Cartesian Plot button. The Big Polar Plot button