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Serial parallel folding with friction: a primitive model using cubic B-splines

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Abstract

An earlier nonlinear model for two-layer parallel folding with bedding plane slip is extended to embrace serial buckling behaviour. Approximating the folds using two cubic B-splines, a quasi-energy formulation admits both synchronous and serial-type buckling under conditions of both controlled load and controlled end-shortening. In the early stages of evolution, non-periodic saddle points, corresponding to localized folds, are found to provide the preferred solution. However, as the end-shortening increases, the saddle points converge with unstable maxima representing synchronous folding, until only periodic solutions exist. This shift from localized to two-hump periodic behaviour is seen as a primitive exposition of the more general theme of serial or sequential fold formation.

Keywords: Layered structures; Localization; Nonlinear response; Buckling; Energy Methods; Instability

1. Introduction

This paper presents a rigorous analysis of serial folding, a process long recognised by geologists as occurring in rocks, but which has not yet been put on a sound mechanical basis. Field observations (Price, 1970, 1975) and analogue experiments (Cobbold, 1975; Blay et al., 1977) have established that serial folding is a common phenomenon in the folding of rocks and it is therefore appropriate to seek a mathematical explanation for this type of behaviour. Fig. 1 shows four stages in an experiment on layers of A4 size paper held together transversely under an applied overburden pressure and compressed in the longitudinal direction by slow application of end displacement to initiate a sequence of parallel folds spreading from the loaded edge. More details can be found in Edmunds et al. (2006). The behaviour shown in Fig. 1 is known variously as sequential amplification (Price and Cosgrove, 1990), serial folding (Blay et al., 1977) or cellular buckling (Hunt et al., 2000). Although the humps form sequentially, it is important to note that the load transfers through the length of the sample without reduction and is not being applied sequentially. The behaviour then differs fundamentally from its spontaneously occurring counterpart (Hunt, 2006), with the resulting wavelengths expected to be different for instance (Budd et al., 2001).

In this paper an analysis is presented for the buckling of a confined multilayer subjected to either a constant external stress or a constant external strain. It complements the work of Biot (1961), Ramberg (1961) and Johnson (1977) in that, unlike these earlier studies, it addresses the problems of the sequential formation of folds rather than spontaneous amplification. Biot (1961) analysed the buckling behaviour of a linear elastic layer set in a viscous matrix. He developed the idea of a dominant wavelength, which amplifies more rapidly than any other, and predicted the formation of a uniform wave-train of sinusoidal buckles. He also studied multilayered elastic models (Biot, 1963, 1964) and reached similar conclusions, namely that a dominant fold wavelength develops and that folding occurs uniformly throughout the model. Ramberg (1961, 1964) analysed the buckling behaviour of both single and multilayered elastic and viscous systems and reached the same conclusions, i.e. that the buckling occurred uniformly throughout the material. These results are in marked contrast to those of the analysis described in this paper, and the buckling behaviour that occurred in the experimental analogue models.

The paper extends a previous study of non-linear folding of a two layer system with bedding plane slip (Budd et al., 2003), and employs the concept of approximating the folding patterns by using cubic B-splines, which allows for two successive humps of different amplitude to be represented and thence can portray primitive forms of both localized and periodic

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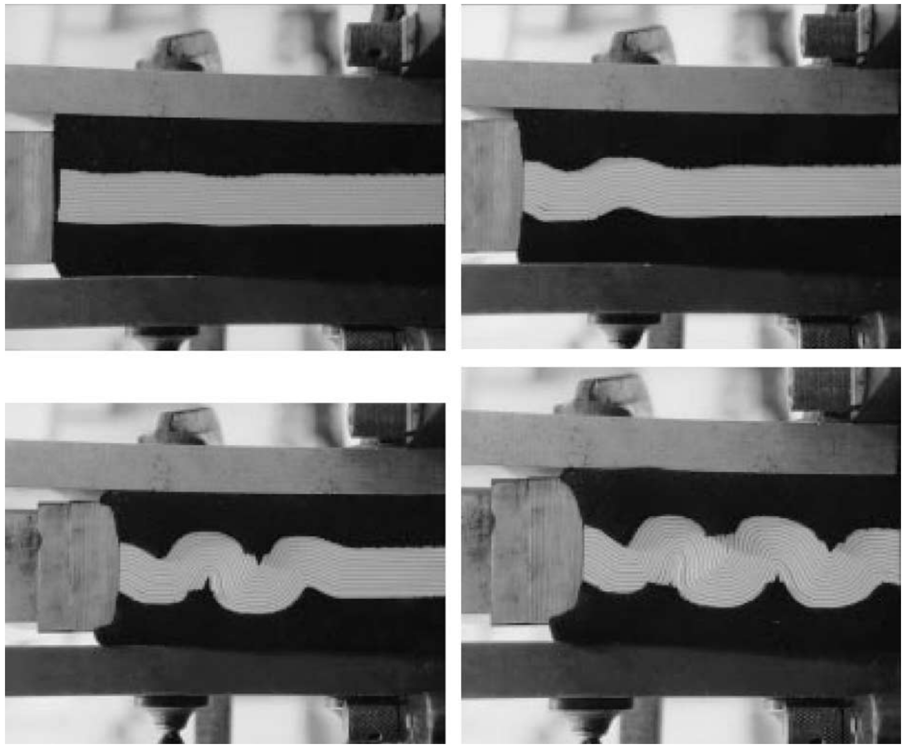

Fig. 1. Parallel folding in layers of paper, showing the serial buckling behaviour.

buckling. We present a rigorous analysis of the process of serial folding in this model, tracking its development and successfully mirroring the sequence seen in Fig. 1, from buckling into a single hump through to the development of the two-hump 'periodic' form as the external loading is changed. Previous work on multilayer buckling by Latham (1985a,b) has confirmed that the localization of folds requires non-linear behaviour, which is built into the model both through the expression for interlayer slip and through a restiffening foundation. The process of bedding plane slip is modelled by allocating frictional properties to the bedding planes owing to the layers being under high overburden pressures (Hobbs et al., 1976). This process is commonly indicated by slickensides, or crystal fibres, on folded bedding planes, and by displaced markers. Remarkably the theory predicts that this slip will occur episodically. Such stick-slip displacement is a process that has been recognized in nature and extensively discussed in the geological literature both with respect to movement along faults and along bedding planes during fold amplification (Hobbs et al., 1976; Price and Cosgrove, 1990). However, this process has not yet been successfully incorporated into any analysis of buckling and we make a systematic derivation in this paper.

Two solutions to the buckling problem are discussed, one relevant to serial folding and the other to spontaneous folding and we show that the type of folding that occurs is controlled by the boundary conditions. Interestingly, the theory confirms that these two folded patterns have different wavelengths.

The problem of the rheology of rocks during folding is also addressed in the paper and the authors agree with previous workers who have argued that elastic buckling solutions are geologically important for predicting fold initiation even when fold amplification is achieved by non-elastic (e.g. viscous or plastic) behaviour. Parallel folds, in particular, are usually found in the upper levels of the Earth's crust, typically in the upper part of an orogenic belt, and this observation supports the use of elasticity theory to study the deformation (de Sitter, 1964).

2. Two layer model

The model is essentially that of two extended elastic beams, held in contact by overburden pressure, but which can slip over each other. If we consider incompressible layers of thickness t, with bending stiffness EI, embedded in a soft foundation of stiffness k per unit length and compressed by a load P (see Fig. 2), and follow classical Euler beam theory, the total

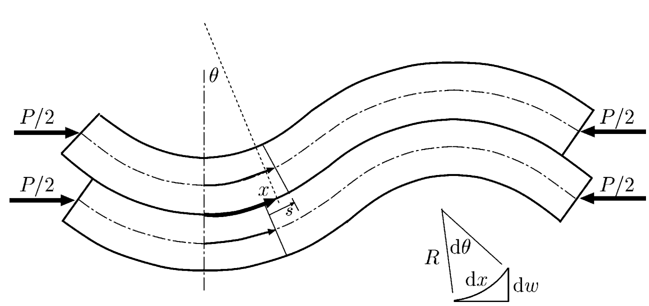

Fig. 2. Slip between incompressible layers constrained to remain in contact.