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The moment method used to infer stress from fault/slip data in sigma space: invalidity and modification

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Abstract

The moment method has recently been used to infer stress in sigma space from fault/slip data. However, if these data are distributed along a hyperplane having a smaller dimension than that of the space minus one, due to limited fault/slip population or biased sampling of it, the best solution of stress vector is not in most cases, as expected, the eigenvector of the datum matrix relating to the smallest eigenvalue. The solution lies within the subspace composed of the eigenvectors relating to the small eigenvalues, for which some auxiliary constraints need to be included. Shear sense constraint alone is adopted, and incorporated by way of grid search, which gives rise to a range of accepted stress vectors in the subspace. Examples from the Chelungpu fault, Taiwan, illustrate the feasibility of the proposed scheme.

Keywords: Stress inversion; Sigma space; Fault/slip data; Moment method; Biased sampling; Indeterminacy

1. Introduction

Although stress inversion appears nonlinear in character, Fry (1999) transformed fault/slip data into datum vectors in 'sigma space' where they tend to be distributed in or near a hyperplane if they were produced in a single tectonic phase. In contrast to some conventional nonlinear schemes (e.g. Angelier, 1984; Xu, 2004), this justifies a new, for the most part linear, scheme for inversion of stress. For single-phase (or homogeneous) data, there is an analytical solution for the 'stress vector' directed normal to the hyperplane. It is the eigenvector corresponding to the fifth largest eigenvalue of the data matrix. (This is the smallest in the 5D space of Shan et al. (2003), but the second smallest of Fry (1999) because he retained the irresolvable (isotropic stress) sigma axis.) This is known as the moment method, as the eigenvector is that of the second moment (or Scheidegger) tensor, composed of second moments—the moments and products of inertia—of the data set. Eigenvector polarity is not constrained in the determination. So, for any solution, its negative is equally valid. Completion of the stress inversion, by discriminating between polarities, requires knowledge of observed fault slip senses, which are not taken into account within the moment method.

It is generally implicit in this scheme that the estimated stress vector should be the unique parameter that best describes the planar distribution of datum vectors in sigma space. Uniqueness requires full dimensionality of the hyperplane of datum vectors of the four dimensions, after conventional 'reduction' to remove indeterminacies (Fry, 1999). Lower dimensionality increases the degrees of freedom of the solution (Fry, 1999).

While recently applying the scheme to real data sets of probably a single phase, we found that this implicit condition is often not fulfilled. Some of them will be discussed below. In these real cases, the hyperplane has smaller dimensionality. So, the stress vector associated with the smallest eigenvector, rather than being a unique solution, is one member of a range of solutions represented by a plane or volume in sigma space. Fry (1999) introduced an unrelated geometric space—'q-space'—in which the distribution of the data through this range could be considered in combination with known shear senses. The aim of this communication is to develop a practical alternative modification, in which treatment of shear sense is integrated into sigma space, rather than subsequent to it.

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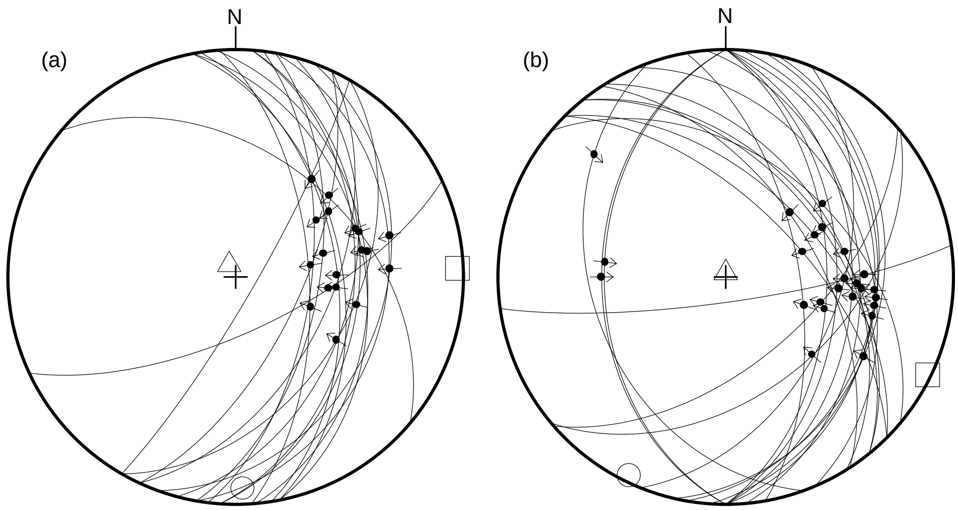

Fig. 1. Equal-area, lower hemispheric projection of fault/slip data measured at segments D (a) and C (b) of the Chelungpu fault, respectively. Fault data (Lee et al., 2002) were provided by Blenkinsop (in press). Unfilled squares, circles and triangles represent the maximum, the intermediate and the minimum principal axes, respectively. They represent the stress tensor restored from the geometric mean values of accepted stress vectors.

2. An example showing the failure of the simple moment method

To show the phenomenon described above, an example is taken from Lee et al. (2003), as modified by Blenkinsop (2006). It consists of 18 fault/slip data (Fig. 1a) from segment D of the active Chelungpu fault, western Taiwan. (See Blenkinsop (in press) for descriptive summary, context and comparative analyses.) These surface rupture data had been produced during the 1999 Chi-Chi earthquake along the Chelungpu fault (Lee et al., 2002, 2003; Angelier et al., 2003). All slip senses were reverse and generally plunging towards the east (Fig. 1). Application of the moment method (Fry, 1999; Shan et al., 2004) to the data set gives rise to results listed in Tables 1 and 2.

In Table 1, all the eigenvectors of the datum matrix are listed in descending order of eigenvalue. The eigenvector relating to the smallest eigenvalue, symbolised as $\mathbf{v}_5$, represents the best solution of stress vector. However, neither the positive vector nor its negative ($-\mathbf{v}_5$) accords with all observed slip senses (Table 2); only 38 and 61% satisfy the real senses of the fault data, respectively. Of their corresponding stress tensors, the maximum principal direction is 175.93° in the former and 76.07° in the latter. They are approximately perpendicular to each other. The possibility of two such diverse phases in the data set is not supported by the fact that these data were produced along a single reactivation of the fault during the 1999 Chi-Chi earthquake (Lee et al., 2002). Meanwhile, although measurement errors surely exist, it is very difficult or even impossible for them, in the light of their stochastic nature, to produce the two phases having nearly perpendicular maximum principal directions.

3. Reason for the failure

In seeking a unique solution by the moment method, it is implicitly assumed that the eigenvector of the data matrix relating to the smallest eigenvalue is a unique parameter that describes the hyperplane. This assumption does not hold in cases that, when eigenvalues are taken in decreasing order, give an abrupt reduction to low eigenvalue after less than four of them. In such a case, the hyperplane of data is effectively reduced in dimensions from 4 to 3, 2 or even 1. A mundane reason for such a reduction can be biased sampling of fault data at outcrop. For example, repetitious sampling of a single fault set would make fault data vectors cluster in sigma space, probably reducing the dimension of the hyperplane to 1. A more serious concern is that such a reduction can be an inherent

Table 1
Eigenvalues and corresponding eigenvectors ($\mathbf{v}_i$, $i=1, 2,\ldots,5$) of the data matrix for the example from segment D of the Chelungpu fault. The last row is the geometric mean vector ($\mathbf{v}$) according to the method in this paper, being a compromise linear combination of three selected eigenvectors with coefficients as, $\mathbf{v}=0.146\mathbf{v}_5+0.899\mathbf{v}_4+0.412\mathbf{v}_3$. See the text for more explanation

No.	Eigenvalues	Eigenvectors					
		Symbols	σ_{11}	σ_{22}	σ_{12}	σ_{13}	σ_{23}
1	12.8616	$\mathbf{v}_1$	0.093	0.159	−0.696	0.068	0.691
2	3.6694	$\mathbf{v}_2$	−0.221	0.638	−0.097	0.676	−0.281
3	0.8632	$\mathbf{v}_3$	0.388	0.471	0.631	−0.029	0.478
4	0.5538	$\mathbf{v}_4$	0.864	0.080	−0.277	−0.004	−0.414
5	0.0516	$\mathbf{v}_5$	0.215	−0.583	0.177	0.733	0.211
6		$\mathbf{v}$	0.968	0.181	0.037	0.091	−0.144