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Topological structure of fuzzy soft sets

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ABSTRACT

In this paper, the concept of fuzzy soft topology is introduced and some of its structural properties such as neighborhood of a fuzzy soft set, interior fuzzy soft set, fuzzy soft basis, fuzzy soft subspace topology are studied.

1. Introduction

We are not able to use classical methods to solve some kind of problems given in sociology, economics, environment, engineering etc., since, these kinds of problems have their own uncertainties. Fuzzy set theory, which was firstly proposed by researcher Zadeh [1] in 1965, has become a very important tool to solve these kinds of problems and provides an appropriate framework for representing vague concepts by allowing partial membership. Fuzzy set theory has been studied by both mathematicians and computer scientists and many applications of fuzzy set theory have arisen over the years, such as fuzzy control systems, fuzzy automata, fuzzy logic, fuzzy topology etc. Beside this theory, there are also theory of probability, rough set theory which deal with to solve these problems. Each of these theories has its inherent difficulties as pointed out in 1999 by Molodtsov [2] who introduced the concept of soft set theory which is a completely new approach for modeling uncertainty. In this paper, Molodtsov established the fundamental results of this new theory and successfully applied the soft set theory into several directions, such as smoothness of functions, operations research, Riemann integration, game theory, theory of probability and so on. Maji et al. [3] defined and studied several basic notions of soft set theory in 2003. Pei and Miao [4], Feng et al. [5], Chen et al. [6], Aktaş and Çağman [7], and Irfan Ali et al. [8] improved the work of Maji et al. [3].

Maji et al. [9] initiated the study involving both fuzzy sets and soft sets. In this paper, the notion of fuzzy soft sets was introduced as a fuzzy generalizations of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then many scientists such as X. Yang et al. [10] improved the concept of fuzziness of soft sets.

In [4], it is pointed out that information systems have been studied by many scientists from several domains and that there exist some compact connections between soft sets and information systems. It is also shown in [4] that soft sets are a class of special information systems, called fuzzy information systems, and that research on soft sets and information systems can be unified. Furthermore, some new results and methods can be expected from this unifying. In [11], Kharal and Ahmad defined the notion of a mapping on classes of fuzzy soft sets, which is of fundamental importance in fuzzy soft set theory, to improve this work and they studied the properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets.

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Since topology depends on the ideas of set theory, we introduce the concept of "fuzzy soft topology" using the fuzzy soft sets and give the basic notions of it by following the Chang [12], Kelley [13] and Munkres [14]. This paper may be the starting point for the studies on "fuzzy soft topology", and all results deducted from this paper can be used in the theory of information systems.

2. Preliminaries

Throughout this paper U denotes initial universe, E denotes the set of all possible parameters which are attributes, characteristic or properties of the objects in U, and the set of all subsets of U will be denoted by $\mathcal{P}(U)$.

Definition 2.1 (*[12]*). A fuzzy set A in U is defined by a membership function $\mu_A : U \longrightarrow [0, 1]$ whose membership value $\mu_A(x)$ specifies the degree to which $x \in U$ belongs to the fuzzy set A, for $x \in U$.

The family of all fuzzy sets in U will denote by $\mathcal{F}(U)$. If $A, B \in \mathcal{F}(U)$ then some basic fuzzy set operations are given componentwise proposed by Zadeh [1] as follows:

(1) $A \subseteq B \Longleftrightarrow \mu_A(x) \leq \mu_B(x)$, for all $x \in U$.
(2) $A = B \Longleftrightarrow \mu_A(x) = \mu_B(x)$, for all $x \in U$.
(3) $C = A \cup B \Longleftrightarrow \mu_C(x) = \mu_A(x) \vee \mu_B(x)$, for all $x \in U$.
(4) $D = A \cap B \Longleftrightarrow \mu_D(x) = \mu_A(x) \wedge \mu_B(x)$, for all $x \in U$.
(5) $E = A^c \Longleftrightarrow \mu_E(x) = 1 - \mu_A(x)$, for all $x \in U$.

Definition 2.2 (*[2]*). Let A be a subset of E. A pair (F, A) is called a soft set over U where $F : A \longrightarrow \mathcal{P}(U)$ is a set-valued function.

As mentioned in [3], a soft set (F, A) can be viewed $(F, A) = \{a = F(a) \mid a \in A\}$ where the symbol "$a = F(a)$" indicates that the approximation for $a \in A$ is $F(a)$.

Definition 2.3 (*[9]*). Let $A \subset E$ and $\mathcal{F}(U)$ be the set of all fuzzy sets in U. Then a pair (f, A) is called a fuzzy soft set over U, where $f : A \longrightarrow \mathcal{F}(U)$ is a function.

From the definition, it is clear that $f(a)$ is a fuzzy set in U, for each $a \in A$, and we will denote the membership function of $f(a)$ by $f_a : U \longrightarrow [0, 1]$.

Similar to the viewing a soft set, a fuzzy soft set (f, A) can be viewed $(f, A) = \{a = \{u_{f_a(u)} \mid u \in U\} \mid a \in A\}$ where the symbol "$a = \{u_{f_a(u)} \mid u \in U\}$" indicates that the membership degree of the element $u \in U$ is $f_a(u)$ where $f_a : U \longrightarrow [0, 1]$ is the membership function of the fuzzy set $f(a)$ [16].

Example 2.4. Suppose that U is the set of houses under consideration, E is the set of parameters where each parameter is a fuzzy word or sentence involving fuzzy words, $E = \{$expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, in bad repair$\}$. In this case, to define soft set and fuzzy soft set means to point expensive houses, beautiful houses, and so on. The soft set (F, A) and the fuzzy soft set (f, A) describes the attractiveness of houses. Suppose that, there are six houses in the universe U given by $U = \{h^1, h^2, h^3, h^4, h^5, h^6\}$ and $A = \{e_1, e_2, e_3, e_4, e_5\} \subset E$ where e_1 stands for the parameter 'expensive', e_2 stands for the parameter 'beautiful', e_3 stands for the parameter 'wooden', e_4 stands for the parameter 'cheap', e_5 stands for the parameter 'in the green surroundings'.

From Definition 2.2, $(F, A) = \{e_1 = \{h^2, h^4\}, e_2 = \{h^1, h^3\}, e_3 = \{h^3, h^4, h^5\}, e_4 = \{h^1, h^3, h^5\}, e_5 = \{h^1\}\}$ is a soft set over U.

From Definition 2.3, $(f, A) = \{e_1 = \{h^1_{0.5}, h^2_1, h^3_{0.4}, h^4_1, h^5_{0.3}, h^6_0\}, e_2 = \{h^1_1, h^2_{0.4}, h^3_1, h^4_{0.4}, h^5_{0.6}, h^6_{0.8}\}, e_3 = \{h^1_{0.2}, h^2_{0.3}, h^3_1, h^4_1, h^5_1, h^6_0\}, e_4 = \{h^1_1, h^2_0, h^3_1, h^4_{0.2}, h^5_1, h^6_{0.2}\}, e_5 = \{h^1_1, h^2_{0.1}, h^3_{0.5}, h^4_{0.3}, h^5_{0.2}, h^6_{0.3}\}\}$ is a fuzzy soft set over U.

Definition 2.5 (*[15]*). Let $A, B \subset E$ and (f, A), (g, B) be two fuzzy soft sets over a common universe U. We say that (f, A) is fuzzy soft subset of (g, B) and write $(f, A) \tilde{\subset} (g, B)$ if and only if

(i) $A \subset B$,
(ii) for each $a \in A$, $f_a(x) \leq g_a(x)$, $\forall x \in U$.

Definition 2.6 (*[15]*). Let $A, B \subset E$. We say that the fuzzy soft sets (f, A) and (g, B) are equal if and only if $(f, A) \tilde{\subset} (g, B)$ and $(g, B) \tilde{\subset} (f, A)$.

Definition 2.7 (*[15]*). Union of two fuzzy soft sets (f, A) and (g, B) over a common universe U is the fuzzy soft set (h, C), which is denoted by $(h, C) = (f, A) \tilde{\cup} (g, B)$, where $C = A \cup B$ and for each $c \in C$,

$$h_c(x) = \begin{cases} f_c(x), & \text{if } c \in A - B \\ g_c(x), & \text{if } c \in B - A \\ f_c(x) \vee g_c(x), & \text{if } c \in A \cap B \end{cases}, \forall x \in U.$$