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A modified symmetric successive overrelaxation method for augmented systems

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ABSTRACT

In this paper, we establish a modified symmetric successive overrelaxation (MSSOR) method, to solve augmented systems of linear equations, which uses two relaxation parameters. This method is an extension of the symmetric SOR (SSOR) iterative method. The convergence of the MSSOR method for augmented systems is studied. Numerical examples show that the new method is an efficient method.

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1. Introduction

Consider the following augmented system:

$$\begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ q \end{pmatrix}, \tag{1}$$

where A is an $m \times m$ real symmetric and positive definite matrix and B is an $m \times n$ real matrix. System (1) can be found in many different applications of scientific computing, e.g., finite element discretization to solve partial differential equations including Stokes equations and Navier–Stokes equations, constrained least squares problems, and generalized least squares problems (see [1–6]). Such systems typically result from mixed or hybrid finite element approximations of second-order elliptic problems, elasticity problems or the Stokes equations [7], and from Lagrange multiplier methods [8].

Some researchers have investigated various methods to solve augmented systems such as (1). Golub et al. [9] presented several SOR-like algorithms. Bai et al. [10] also developed a SOR-like method, and presented the generalized SOR (GSOR) method for augmented linear systems. Darvishi and Hessari [11] developed the symmetric SOR (SSOR) iterative method to solve such systems. Also, Zhang and Lu [12] presented the generalized symmetric SOR (GSSOR) method for solving large sparse augmented systems. One may find details of symmetric iterative methods to solve different kinds of linear systems in [13–17].

In this paper, we develop a modified symmetric SOR method to solve augmented system (1) using two relaxation parameters.

The rest of the paper is organized as follows. In Section 2, the outline of the modified symmetric SOR method to solve (1) is provided. In Section 3, we obtain the convergence region for this method. In Section 4, some numerical examples are presented. Finally, the paper is concluded in Section 5.

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2. The modified symmetric SOR (MSSOR) method

For the sake of simplicity, we rewrite augmented linear system (1) as

$$\begin{pmatrix} A & B \\ -B^T & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ -q \end{pmatrix}, \tag{2}$$

where A is an $m \times m$ real symmetric and positive definite matrix and B is an $m \times n$ real matrix. Because of zero block in the coefficient matrix, we cannot solve the system by the SOR method. Hence, Golub et al. [9] presented several SOR-like algorithms to solve such an augmented system. In this paper, for the coefficient matrix of augmented system (2), we consider the following splitting:

$$\mathcal{A} = \begin{pmatrix} A & 0 \\ 0 & Q \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ B^T & \alpha Q \end{pmatrix} - \begin{pmatrix} 0 & -B \\ 0 & \beta Q \end{pmatrix} = \mathcal{D} - \mathcal{A}_l - \mathcal{A}_u, \tag{3}$$

where Q is an $n \times n$ real symmetric and nonsingular matrix, and α and β are two real numbers satisfying $\alpha + \beta = 1$ with $\alpha\beta \neq 0$.

Let

$$\mathcal{L} = \mathcal{D}^{-1}\mathcal{A}_l = \begin{pmatrix} 0 & 0 \\ Q^{-1}B^T & \alpha I \end{pmatrix}, \qquad \mathcal{U} = \mathcal{D}^{-1}\mathcal{A}_u = \begin{pmatrix} 0 & -A^{-1}B \\ 0 & \beta I \end{pmatrix}.$$

Let $\mathbf{z}^{(k)} = \begin{pmatrix} x^{(k)} \\ y^{(k)} \end{pmatrix}$ be the kth approximation of solution (2) by the modified symmetric SOR method using splitting (3). Symmetric successive overrelaxation methods can be considered to be a double-sweep method. The first sweep is in fact the special method itself, while the second one is the iterative method with the roles of lower and upper triangular matrices in the iterative method interchanged. By deleting a common vector in two relations, one can obtain a single equation; this is called a symmetric iterative method. In the modified symmetric SOR method, first we obtain $\mathbf{z}^{(k+\frac{1}{2})}$ from $\mathbf{z}^{(k)}$ by the forward SSOR method [11] as follows:

$$\mathbf{z}^{(k+\frac{1}{2})} = \mathcal{J}_1\mathbf{z}^{(k)} + (\mathcal{I} - \Omega\mathcal{L})^{-1}\Omega\mathcal{D}^{-1}\mathbf{c}, \tag{4}$$

where

$$\Omega = \begin{pmatrix} \omega I_m & 0 \\ 0 & \tau I_n \end{pmatrix},$$

$$\begin{aligned} \mathcal{J}_1 &= (\mathcal{I} - \Omega\mathcal{L})^{-1}[(\mathcal{I} - \Omega) + \Omega\mathcal{U}] \\ &= \begin{pmatrix} (1-\omega)I_m & -\omega A^{-1}B \\ \dfrac{\tau(1-\omega)}{1-\alpha\tau}Q^{-1}B^T & \dfrac{1-\tau+\beta\tau}{1-\alpha\tau}I_n - \dfrac{\omega\tau}{1-\alpha\tau}Q^{-1}B^TA^{-1}B \end{pmatrix} \end{aligned}$$

and

$$\mathbf{c} = \begin{pmatrix} b \\ -q \end{pmatrix}.$$

Note that

$$\mathcal{I} - \Omega\mathcal{L} = \begin{pmatrix} I_m & 0 \\ -\tau Q^{-1}B^T & (1-\alpha\tau)I_n \end{pmatrix},$$

so we have $\det(\mathcal{I} - \Omega\mathcal{L}) = (1-\alpha\tau)^n \neq 0$ if and only if $1 - \alpha\tau \neq 0$.

Then, by the backward SSOR method, we compute $\mathbf{z}^{(k+1)}$ from $\mathbf{z}^{(k+\frac{1}{2})}$ as

$$\mathbf{z}^{(k+1)} = \mathcal{J}_2\mathbf{z}^{(k+\frac{1}{2})} + (\mathcal{I} - \Omega\mathcal{U})^{-1}\Omega\mathcal{D}^{-1}\mathbf{c}, \tag{5}$$

where

$$\begin{aligned} \mathcal{J}_2 &= (\mathcal{I} - \Omega\mathcal{U})^{-1}[(\mathcal{I} - \Omega) + \Omega\mathcal{L}] \\ &= \begin{pmatrix} (1-\omega)I_m - \dfrac{\omega\tau}{1-\beta\tau}A^{-1}BQ^{-1}B^T & -\dfrac{\omega(1-\tau+\alpha\tau)}{1-\beta\tau}A^{-1}B \\ \dfrac{\tau}{1-\beta\tau}Q^{-1}B^T & \dfrac{1-\tau+\alpha\tau}{1-\beta\tau}I_n \end{pmatrix}. \end{aligned}$$

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